Question One

$$M\ddot{y} + Ky = F_1\cos\omega t - F_2\cos(\omega_n t)$$

$$\omega_n = \sqrt{\frac{K}{M}}$$ ← Resonance (natural) frequency

$y(0) = 0$

$\dot{y}(0) = 0$

$$\ddot{y} + \omega_n^2 y = \underbrace{\frac{F_1}{M}\cos\omega t}_{y_{p1}} - \underbrace{\frac{F_2}{M}\cos(\omega_n t)}_{y_{p2}}$$

* different Angles so need y_{p2}

Step one: y_h

*underdamped

$a = 1$

$b = 0$

$c = \omega_n^2$

$$\frac{-0 \pm \sqrt{0^2 - 4(1)(\omega_n)^2}}{2(1)}$$

$$\frac{0 \pm \sqrt{-4\omega_n^2}}{2}$$

$$y_h = e^{0t}(A\cos\omega_n t + B\sin\omega_n t)$$

$$\frac{0 \pm i\sqrt{4\omega_n^2}}{2}$$

$$\frac{2\omega_n i}{2}$$

$$\lambda = 0 \pm \omega_n i$$

Step Two: y_{p1} (No Dupe) $\quad y_h = e^{0t}(A\cos\omega_n t + B\sin\omega_n t)$

$$\ddot{y} + \omega_n^2 y = \frac{F_1}{m}\cos\omega t$$

$$= \frac{F_1}{m} RE[e^{i\omega t}]$$

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^{i\omega t} = \cos\omega t + i\sin\omega t$

$= RE[e^{i\omega t}]$

$$\ddot{Y} + \omega_n^2 Y = \frac{F_1}{m} e^{i\omega t}$$

$$Y_p = De^{i\omega t}$$

$y_p = RE[Y_p]$

$$\dot{Y}_p = De^{i\omega t}[i\omega + 0]$$

$$\ddot{Y}_p = -De^{i\omega t}[\omega^2]$$

$$-De^{i\omega t}[\omega^2] + \omega_n^2(De^{i\omega t}) = \frac{F_1}{m} e^{i\omega t}$$

$$D\cancel{e^{i\omega t}}(-\omega^2 + \omega_n^2) = \frac{F_1}{m}\cancel{e^{i\omega t}}$$

$$D = \frac{F_1}{m(-\omega^2 + \omega_n^2)}$$

$$Y_{p1} = \frac{F_1}{m(-\omega^2 + \omega_n^2)} e^{i\omega t}$$

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$$e^{i\theta} = \underbrace{\cos\theta}_{RE} + i\sin\theta$$

$$y_{p_1} = RE[Y_{p_1}]$$

$$e^{i\omega t} = \underbrace{\cos\omega t} + i\sin\omega t$$

$$Y_{p1} = \frac{F_1}{m(-\omega^2 + \omega_n^2)}(\cos\omega t + i\sin\omega t)$$

$$y_{p1} = \frac{F_1 \cos\omega t}{m(-\omega^2 + \omega_n^2}$$

Dupcs

$$\ddot{y} + \omega_n^2 y = -\frac{F_z}{m}\cos(\omega_n t)$$

$$y_p = RE[Y_p]$$

$$\ddot{Y} + \omega_n^2 Y = \frac{F_z}{m} e^{i\omega_n t}$$

$$Y_p = D t e^{i\omega_n t}$$

$$\dot{Y}_p = De^{i\omega_n t}(i\omega_n (t) + 1)$$

$$\ddot{Y}_p = De^{i\omega_n t}(i\omega_n(i\omega_n(t) + 1) + i\omega_n)$$

$$-\omega_n^2(t) + 2i\omega_n$$

$$De^{i\omega_n t}(-\omega_n^2(t) + 2i\omega_n) + \omega_n^2 D t e^{i\omega_n t} = -\frac{F_z}{m} e^{i\omega_n t}$$

$$D\cancel{e^{i\omega_n t}}(\cancel{-\omega_n^2 t} + 2i\omega_n + \cancel{\omega_n^2 t}) = -\frac{F_z}{m}\cancel{e^{i\omega_n t}}$$

$$D = \frac{-F_z}{m(2i\omega_n)}$$

$$Y_{p_2} = \frac{-F_z t}{m(2i\omega_n)} e^{i\omega_n t}$$

$$Y_{p_2} = \frac{-F_2 t}{m(2i\omega_n)} e^{i\omega_n t}$$

$$= \frac{-F_2 t}{m(2i\omega_n)} \cdot \frac{i}{i} e^{i\omega_n t}$$

$$= \frac{F_2 t\, i}{2m\omega_n} e^{i\omega_n t}$$

* Want $RE[e^{i\omega_n t}]$

$$= \frac{F_2 t\, i}{2m\omega_n}\left(\cos\omega_n t + i\sin\omega_n t\right)$$

$$y_{p_2} = -\frac{F_2 t}{2m\omega_n}\sin\omega_n t$$

$$y = y_n + y_{p1} + y_{p2}$$

$$y(0) = 0$$

$$\dot{y}(0) = 0$$

$$y = A\cos W_n t + B\sin W_n t + \frac{F_1 \cos Wt}{m(-W^2 + W_n^2)} - \frac{F_2 t}{2mW_n} \sin W_n t$$

$$y(0) = A + 0 + \frac{F_1}{m(-W^2 + W_n^2)} - 0$$

$$A = -\frac{F_1}{m(-W^2 + W_n^2)}$$

$$U'V + UV'$$

$$\frac{F_2(1)}{2mW_n} \sin W_n t + \frac{F_2 t W_n}{2mW_n} \cos W_n t$$

$$\dot{y} = -AW_n \sin W_n t + BW_n \cos W_n t + \frac{-F_1 W \sin Wt}{\cdots\cdots} - \frac{F_2}{2mW_n} \sin W_n t - \frac{F_2 t W_n}{2mW_n} \cos W_n t$$

$$0 + BW_n + 0 - 0 - 0$$

$$BW_n = 0$$

$$B = 0$$

$$y = -\frac{F_1}{m(-W^2+W_n^2)}\cos W_n t + \frac{F_1 \cos Wt}{m(-W^2+W_n^2)} - \frac{F_2 t}{2mW_n}\sin W_n t$$

<u>Take Limit</u>

$$\lim_{W \to W_n}\left(-\frac{F_1 \cos W_n t}{m(-W^2+W_n^2)} + \frac{F_1 \cos Wt}{m(-W^2+W_n^2)} - \frac{F_2 t}{2mW_n}\sin W_n t\right)$$

Doesn't matter

$$\lim_{W \to W_n}\left(\left(\frac{F_1}{m}\right)\left(\frac{-\cos W_n t}{-W^2+W_n^2} + \frac{\cos Wt}{-W^2+W_n^2}\right)\right)$$

(Has constants and W_n)

$$\lim_{W \to W_n}\left(\left(\frac{F_1}{m}\right)\left(\frac{-\cos W_n t + \cos Wt}{-W^2+W_n^2}\right)\right)$$

Apply L'Hopital's Rule with Respect to W

$$\lim_{W \to W_n}\left(\left(\frac{F_1}{m}\right)\left(\frac{-t\sin Wt}{-2W}\right)\right)$$

$$= \left(\frac{F_1}{m}\right)\left(\frac{-t\sin W_n t}{-2W_n}\right) - \frac{F_2 t}{2mW_n}\sin W_n t$$