Homogenous Systems

$\ddot{y}$ = Acceleration
$\dot{y}$ = Velocity

$$m\ddot{y} + cy + ky = 0$$

How to find homogenous y_h

$$y = e^{\lambda t} \quad \begin{array}{|l} \ddot{y} \rightarrow \lambda^2 \\ \dot{y} \rightarrow \lambda \\ y \rightarrow \lambda \end{array}$$

$$\lambda_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

There are three cases:

if $c^2 - 4mk = 0$

if $c^2 - 4mk$ is positive (or $c^2 > 4mk$) Overdamped

if $c^2 - 4mk$ is negative, imaginary numbers (or $c^2 < 4mk$)

① $C^2 > 4MK \longrightarrow$ overdamped

Door

λ_1 and λ_2 would be both real

$$y_h = \underbrace{Ae^{\lambda_1 t}}_{\text{First order}} + Be^{\lambda_2 t}$$

To solve this, we need initial conditions to plug in!

$$\lambda_{1,2} = -\frac{C}{2M}$$

② $C^2 = 4MK \longrightarrow$ Critically damped

Photo Camera

Really Fast Damping!

$$\lambda_{1,2} = -\frac{C}{2M}$$

$$y_h = Ae^{-\frac{C}{2m}t} + Bte^{-\frac{Ct}{2m}}$$

Don't want to duplicate!

③ $c^2 < 4mk \longrightarrow$ Underdamped

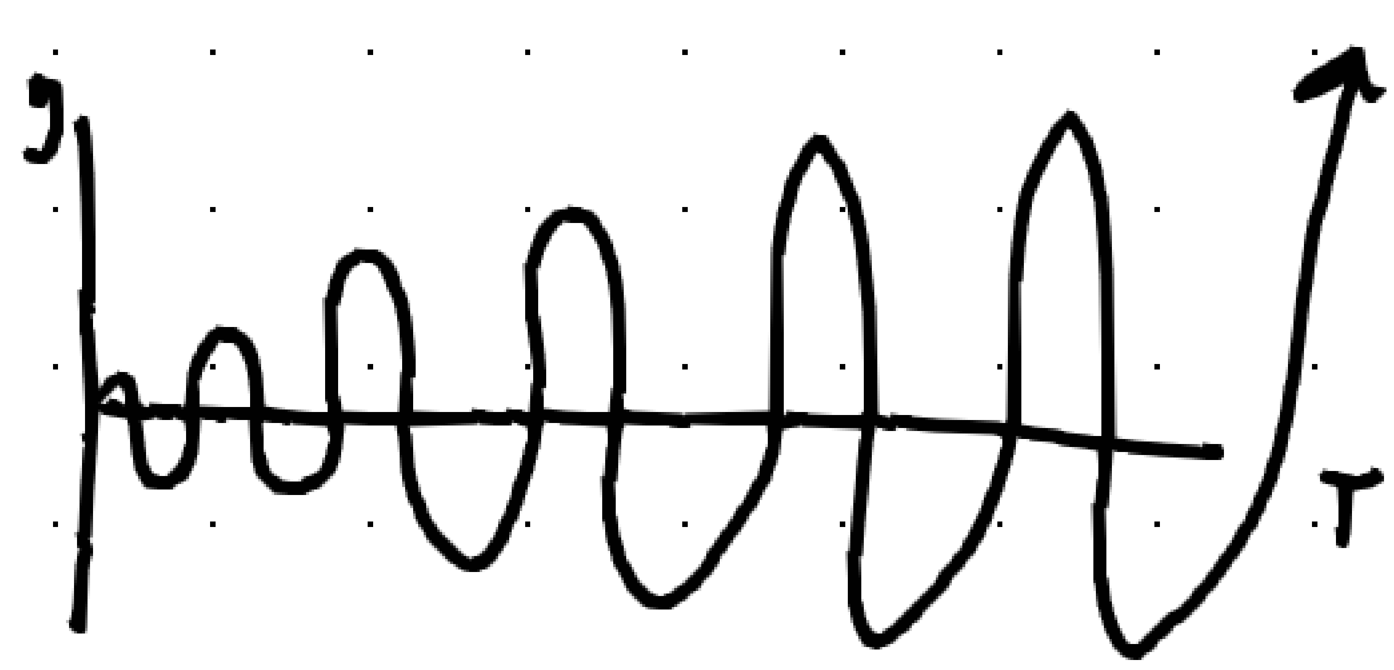
$$y_h = e^{\alpha t}(A\cos\beta t + B\sin\beta t)$$

$$\alpha = -\frac{c}{2m} \qquad \beta = \frac{\sqrt{4mk - c^2}}{2m} \leftarrow \frac{\sqrt{-1}\,\sqrt{c^2 - 4mk}}{2m}$$

c^2 is smaller than $4mk$

$$\lambda_{1,2} = \alpha \pm i\beta$$

Stability $\longrightarrow$ Only dependent on the <u>real part</u> of λ (or Alpha)



← This is an unstable system! y is growing $t \to \infty,\ y \to \infty$

If all λ has negative real part $\longrightarrow$ Stable — Decaying to something constant

$y = 2e^{-3t}$, $\lim_{t\to\infty} y = 2\frac{1}{e^{3t}}\ \infty \to 0$ Stable!

If any λ has positive real part $\longrightarrow$ Unstable — Approaches ∞

If λ has a zero real part $\longrightarrow$ Marginally Stable — Something that doesn't grow or shrink. (Think cos or sin)

↑ Usually No damper.

The Stability/Unstability Comes ONLY from y_h

TLDR:

		Roots	Response
Overdamped	$c^2 > 4mk$	Real, distinct	Slow
Critically Damped	$c^2 = 4mk$	Real, Repeated	Fast
Underdamped	$c^2 < 4mk$	complex conjugates	Osicllatory decay.

Now, we need to take the derivative for the o

$$y_h' = Be^{-4t} + (A+Bt)(-4e^{-4t})$$

Product Rule

$$= e^{-4t}(B-4$$

Examples:

$\to$ C "damper", related to velocity $\dot{y}$

$$\ddot{y} + 8\dot{y} + 16y = 0$$

$y(0) = 0$

$y'(0) = 6$

$\to$ K "spring constant" related to displacement y

$m = 1$

$\ddot{y} = \lambda^2$

$\dot{y} = \lambda$

$y = 1$

$$= \lambda^2 + 8\lambda + 16 = 0$$

Could use quad formula.. but just going to factor.

$$(\lambda + 4)(\lambda + 4) = 0$$

$$\lambda_{1,2} = -4$$

Repeated Root, Critically Damped

$$y_h = Ae^{-4t} + Bte^{-4t}$$

$$\downarrow$$

$$y_h = e^{-4t}(A + Bt)$$

$y(0) = 0 \to \quad 0 = 1(A + B(0))$

1 $A = 0$

Now, we need to take the derivative for the other IC

Product Rule: $u'v + uv'$

$$y_h' = Be^{-4t} + (A + Bt)(-4e^{-4t})$$
$$= e^{-4t}(B-4)(A+Bt)$$

$$y'(t) = Be^{-4t}(1-4t)$$ ← Plug in the Already Solved $A = 0$

Plug in IC

Velocity

$y'(0) = 6$ $\qquad y'(0) = Be^{0}(1-4(0)) = 6$

$\boxed{B = 6}$

So,

$$\boxed{y(t) = 6te^{-4t}}$$

Find the time in which the velocity is zero.

or $y'(t) = 0$

$$y'(t) = 6e^{-4t}(1-4t)$$

$$0 = 6e^{-4t}(1-4t)$$

Solve for t

You can Ignore e!

$$1-4t = 0$$

$$\boxed{t = \frac{1}{4}}$$

e can never be zero, even if t is zero!

Solve Questn Two Theory

Possibly do Three......