Delta functions

Delta is an instantaneous function!

$$\ddot{y} + \dot{y} + 2y = \underbrace{e^{-2t} + t^3}_{f(t)}$$

$\delta(t)$

$$\mathcal{L}\{\delta(t)\} = 1$$

Unit impulse response method (or delta method)

$$\ddot{y} + a_1\dot{y} + a_0 y = f(t)$$

Turn into h's

$$\ddot{h} + a_1\dot{h} + a_0 h = \delta(t)$$

$\xrightarrow{\mathcal{L}}$ ... $H(s) =$ Something

From that, take Laplace inverse $\mathcal{L}^{-1}$ $h(t)$

Once we get $h(t)$, we want to use a Cartan formula

$$y(t) = f(t) * g(t) = \int_0^t f(u)\,g(t-u)\,du = \int_0^t f(t-u)\,g(u)\,du, \quad F(s)G(s)$$

Example of Delta method

$$\ddot{y} + 5\dot{y} + 6y = \underbrace{e^{-3t}}$$

$$\ddot{h} + 5\dot{h} + 6h = f(t) \longrightarrow \delta = 1$$

$$s^2 h(s) - s\cancel{y(0)} - \cancel{\dot{y}(0)} + 5(s\,h(s) - \cancel{y(0)}) + 6h(s) = 1$$

$$0 \qquad 0 \qquad 0$$

$$h(s)(s^2 + 5s + 6) = 1$$

$$h(s) = \frac{1}{(s^2+5s+6)} = \frac{1}{(s+2)(s+3)}$$

$$\frac{1}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{s+}$$

$$1 = A(s+3) + B(s+2)$$

Cover up:

$A = 1$

$B = -1$

$$H(s) = \frac{1}{(s+2)} - \frac{1}{(s+3)}$$

$$\boxed{h(t) = e^{-2t} - e^{-3t}}$$

Apply Formula $y(t) = f(t) * g(t) = \int_0^t f(u)\,g(t-u)\,du = \int_0^t f(t-u)\,g(u)\,du, \quad F(s)G(s)$

$$y(t) = \int_0^t f(u)\,h(t-u)\,du = \int_0^t (e^{-3u})(e^{-2(t-u)} - e^{-3(t-u)})\,du$$