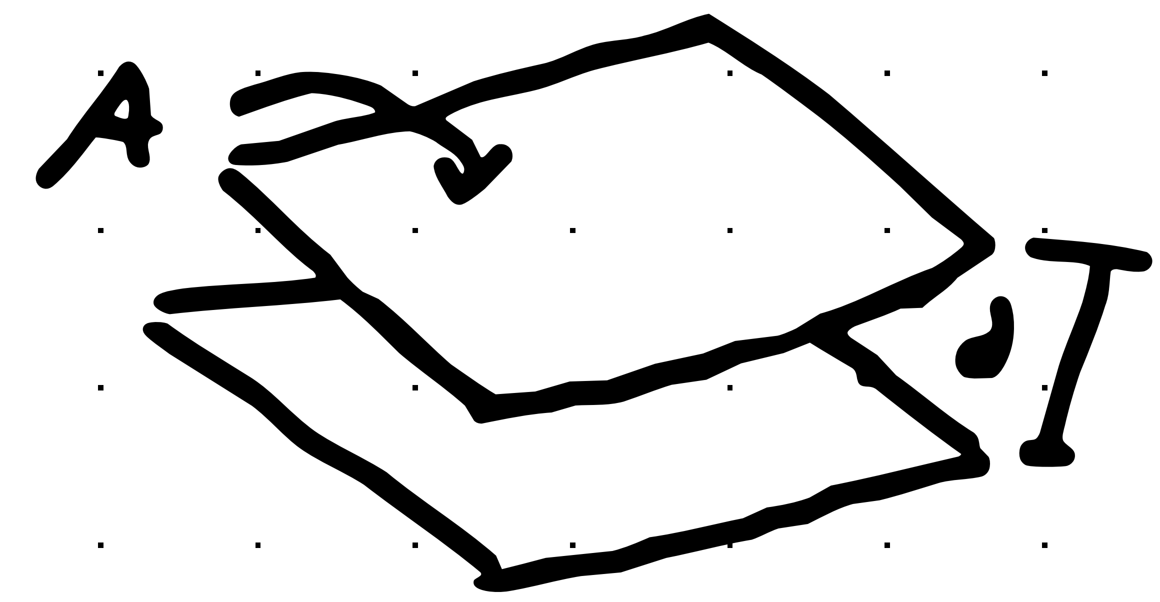
Lecture 16

ECED 2□□□

Capacitors



$$\text{Capacitance} = C = \frac{q}{v}$$

$q \rightarrow$ Charge

$v \rightarrow$ Voltage

(C/V) or (Farad)
F

Permittivity of free space $= 8.84 \times 10^{-12}$ F/m

$$C = \frac{\epsilon A}{d} = \frac{q}{v}$$

A = plate Area

d = distance between two plates

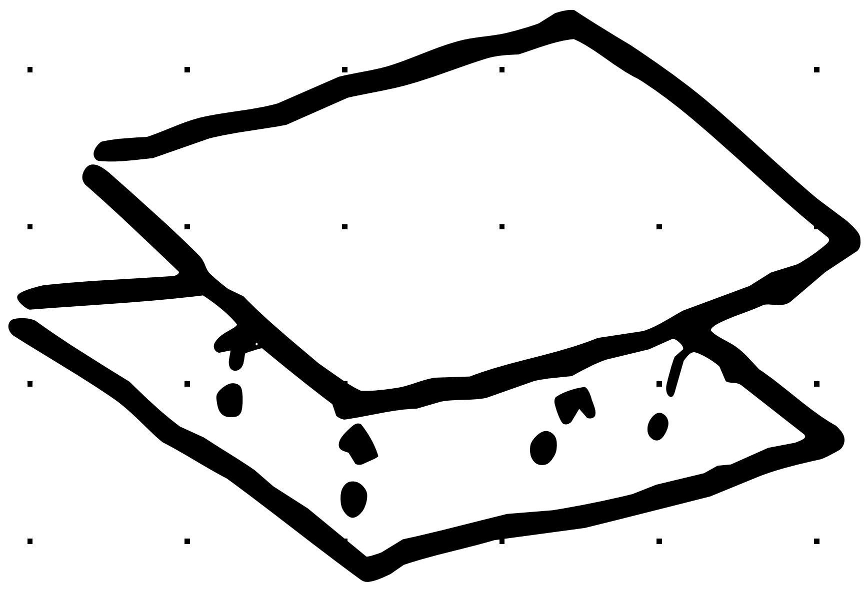
$$i = \frac{dq}{dt} = \frac{d(Cv)}{dt} = C\frac{dv}{dt}$$

Here, we are assuming that the capacitance (C) is constant. This is not true in reality, because the plate distance can change, or the plate media can change. However, for cases, we will assume C is constant.

With Capacitors, Current Leads Voltage

by 90°!

- This is because, in a capacitor, you have two plates that are separated.



- To create a potential difference in between the two plates, you require a bit of current!

- In other words, in order to create voltage, you **NEED** current first!

EQUATIONS USED BY CAPACITORS

$$\boxed{V_{(t)} = \frac{1}{C}\int_{t_0}^{t_1} i_{(t)}\,dt + V_{(t_0)}}$$

Voltage

$$\boxed{i = \frac{dq}{dt} = \frac{d(Cv)}{dt} = C\frac{dv}{dt}}$$

Current

$$\boxed{P = V_{(t)}\, i_{(t)} = C v_{(t)} \frac{dv_{(t)}}{dt}}$$

Power

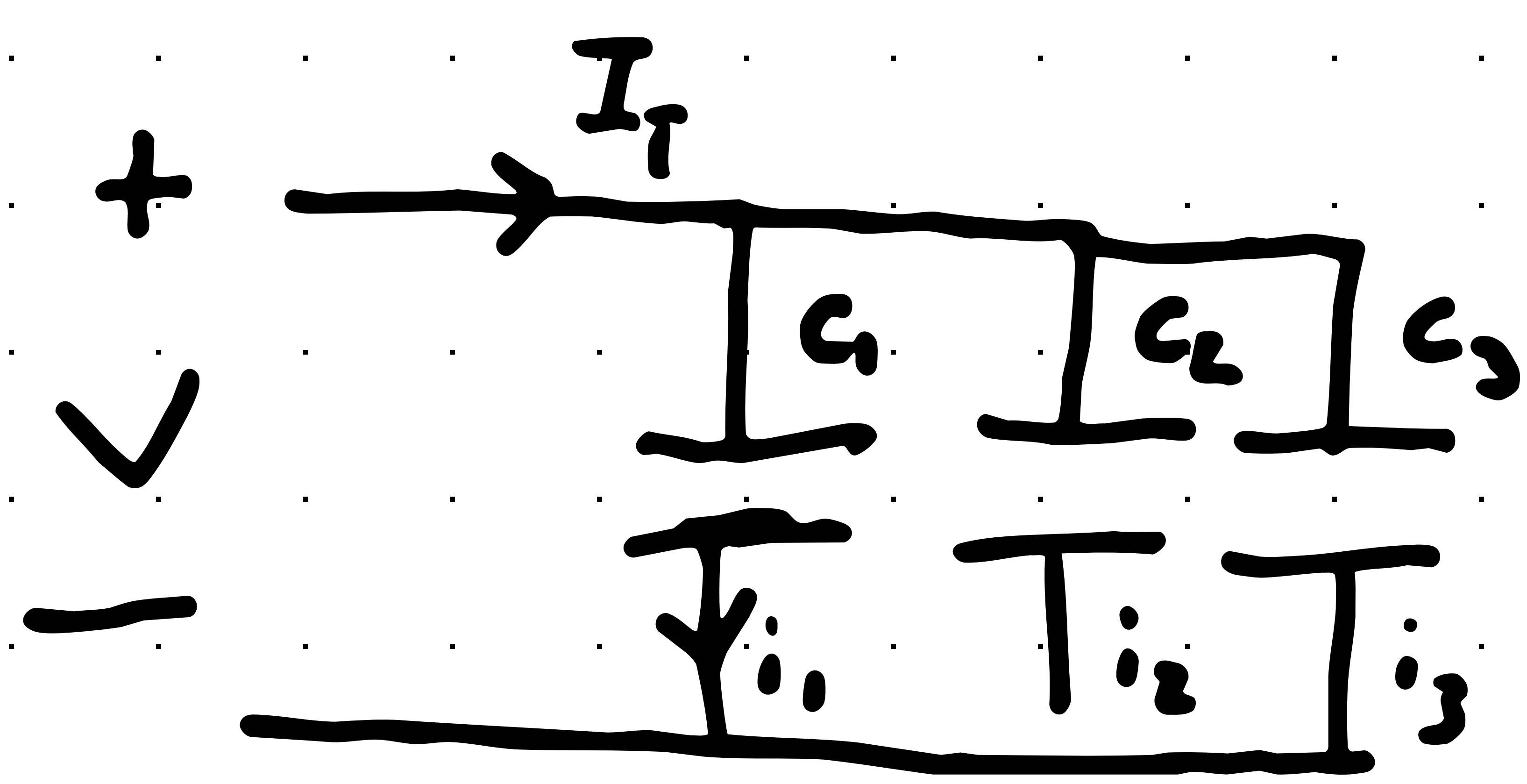
$$\boxed{W \cong \frac{1}{2}CV^2}$$

Energy "stored" in capacitor, or Energy discharged by capacitor

Capacitors In Series

$$\boxed{\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Capacitors In Parallel



$$\boxed{C_T = C_1 + C_2 + C_3}$$

For DC Circuits

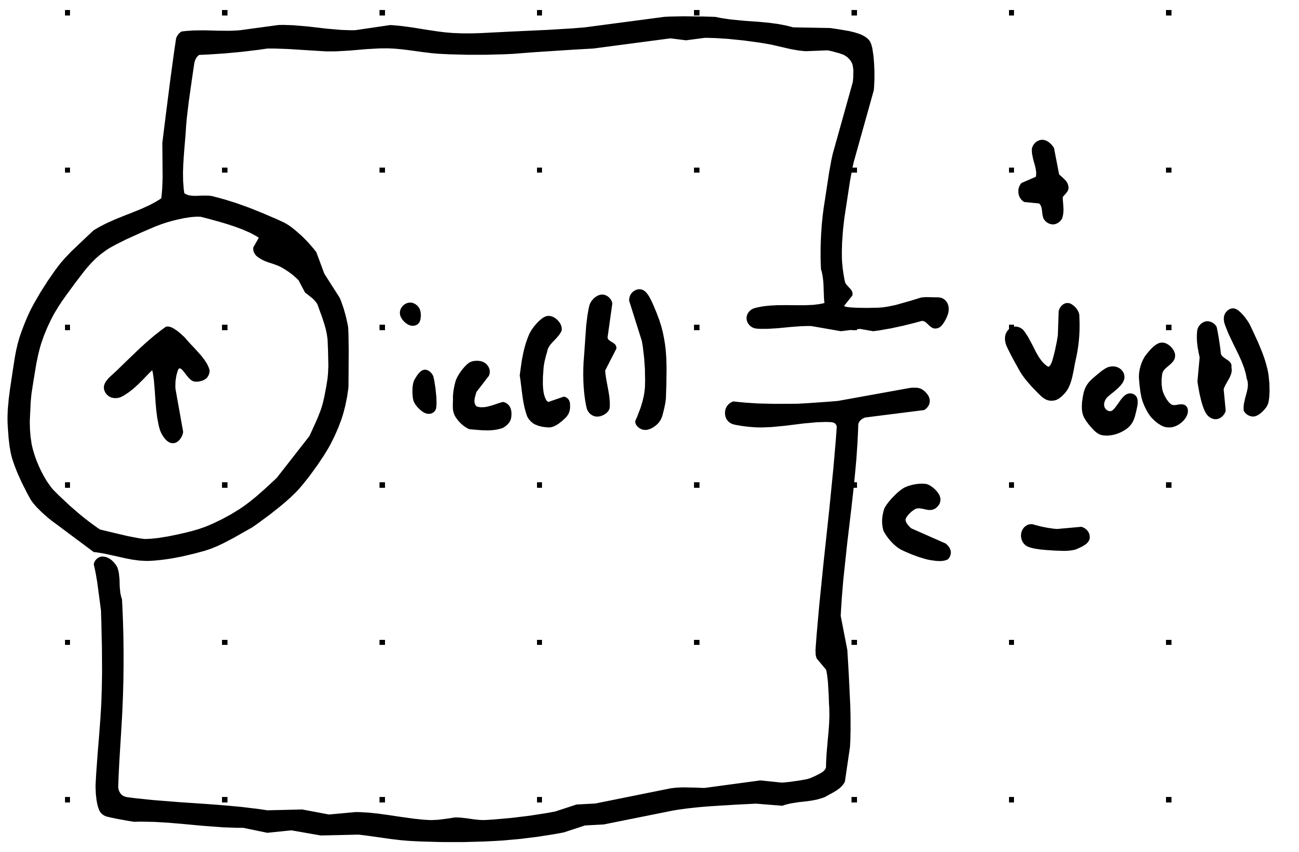
Since $i = C\frac{dv}{dt}$

In DC, the capacitor appears to be an open circuit

This occurs while the capacitor is <u>Charging</u>
This is called the Transient state, or the Transient period. It is not a steady state.
While it "spools" up, there is a period where things are unknown, or unpredictable. It's like when you first get up in the morning. Or, like an old elevator first moving up, right?

Ex

$$i_c(t) = \begin{cases} 0 & t<0 \\ 5000t & 0<t<20\,\mu s \\ 0.2-5000t & 20\mu<t<40\,\mu s \\ 0 & t>0 \end{cases}$$

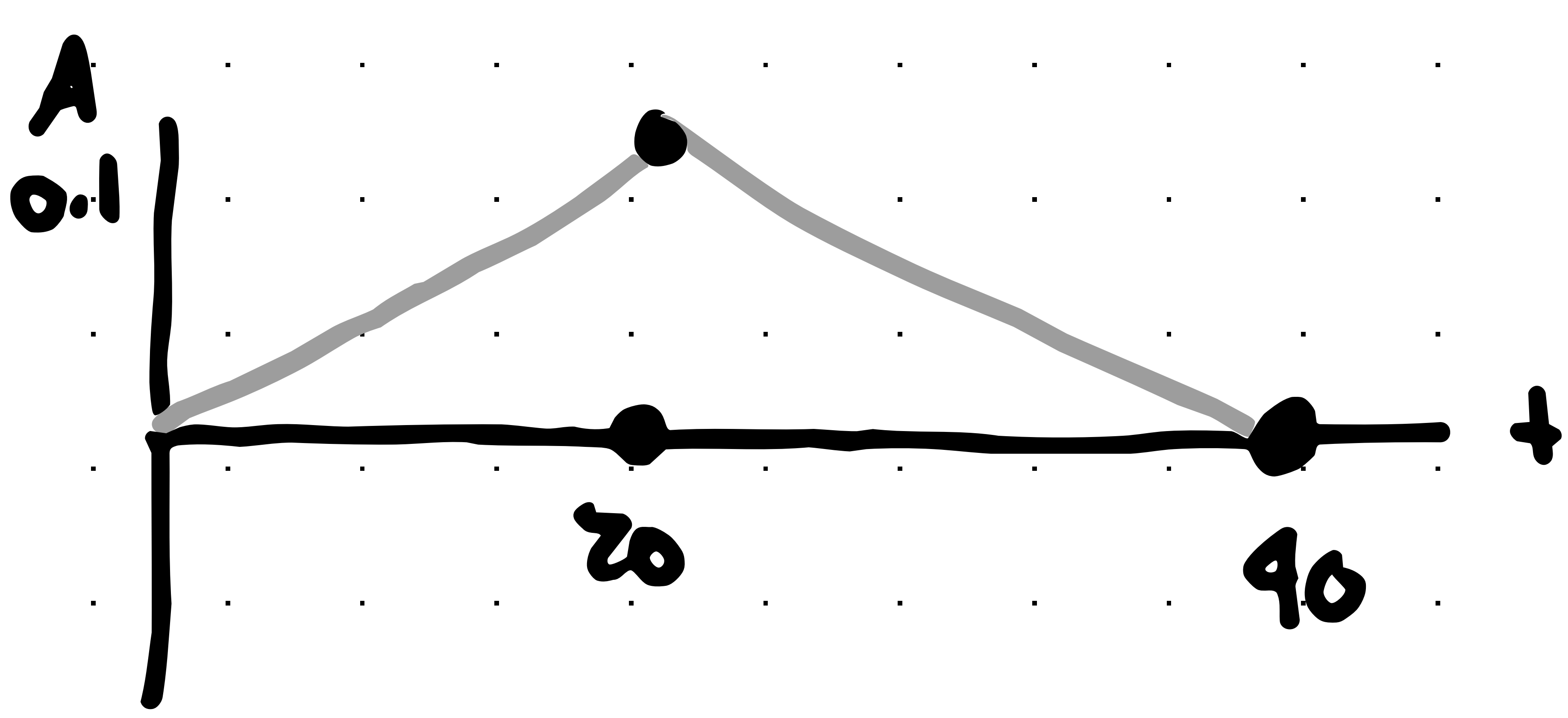


$C = 0.2\,\mu F$

$V_c(0) = 0$

Calculate $V_c(t)$, $P(t)$, $W(t)$ and sketch

Sol



$$V(t) = \frac{1}{C}\int i(t)\,dt + V_0, \quad 0<t<20\,\mu s$$

$$V(t) = \frac{10^6}{0.2}\int_0^t 5000t\,dt + \cancel{V_c(0)}^{\,0}$$

$$V(t) = \frac{10^6}{0.2}\left[5000\frac{t^2}{2}\Big|_0^t\right] = \boxed{12.5\times10^9 t^2}$$

at $t = 20\,\mu s \rightarrow V(t) = 5v$

$20 < t < 40\,\mu s$

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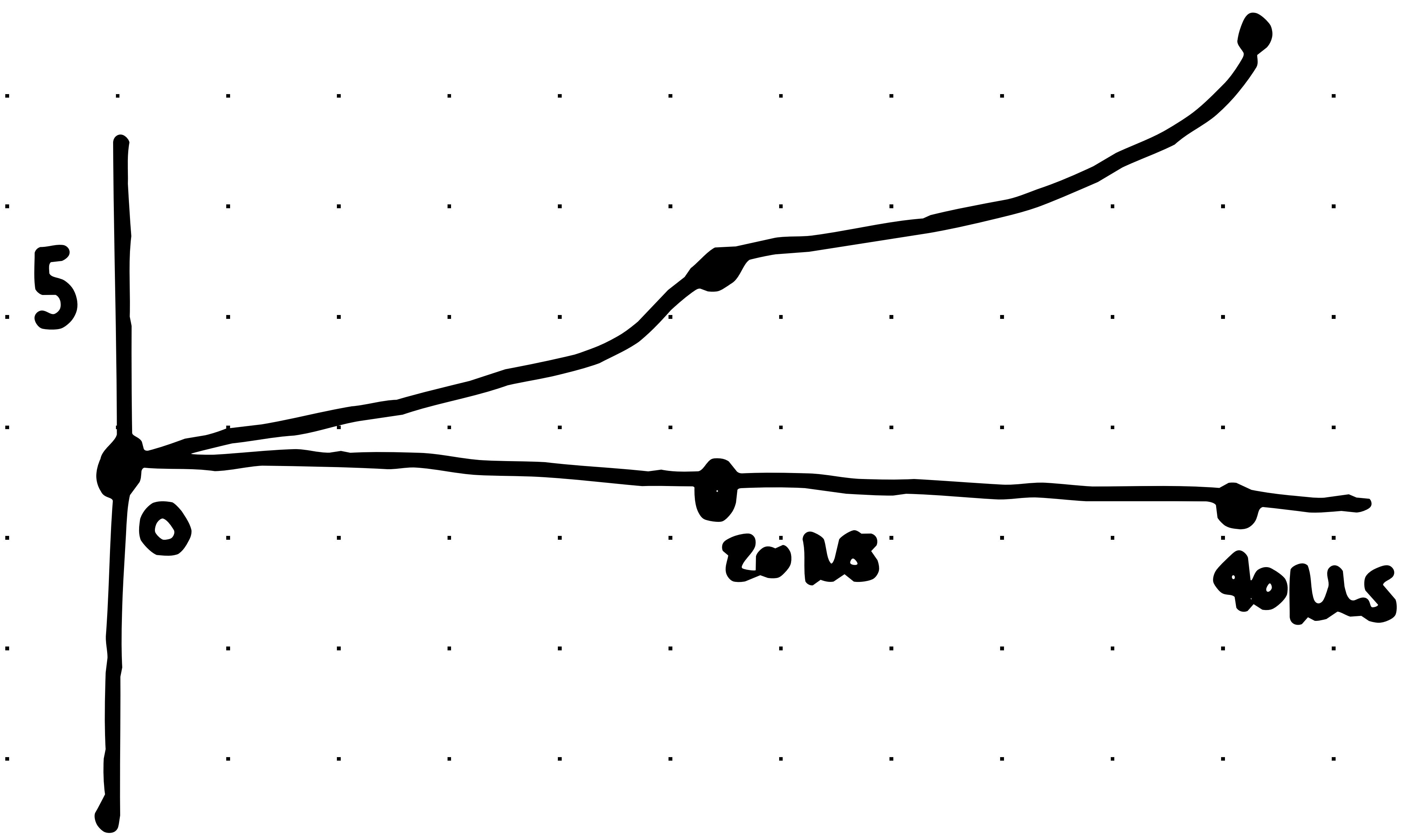
$$V(t) = \frac{10^6}{0.2}\int_{20}^{t}(0.2 - 5000t)\,dt + V(t = 20\,\mu s)$$

$$V(t) = 10^6 t - 12.5\times10^9 t^2 - 10$$



$P = V(t)\, i(t)$

$0 < t < 20\,\mu s$

$W = \int P\,dt = \frac{1}{2}CV^2$

$P(t) = (5000t)(12.5\times10^9 t^2) = 62.5\times10^{12}t^3$

$W = \int (62.5\times10^{12}\,dt$

$= 15.625\times10^{12}\,t$

$20 < t < 40 \mu s$

$$P(t) = (0.2 - 5000t)(10^6 t - 12.5 \times 10^9 t^2 - 10) =$$
$$= 62.5 \times 10^{12} t^3 - 7.5 \times 10^9 t^2 + 2.5 \times 10^5 t - 2$$

$$W(t) = 15.675 \times 10^{12} t^4 - 2.5 \times 10^9 t^3$$
$$+ 0.125 \times 10^6 t^2 - 2 \times 10^{-15} t$$