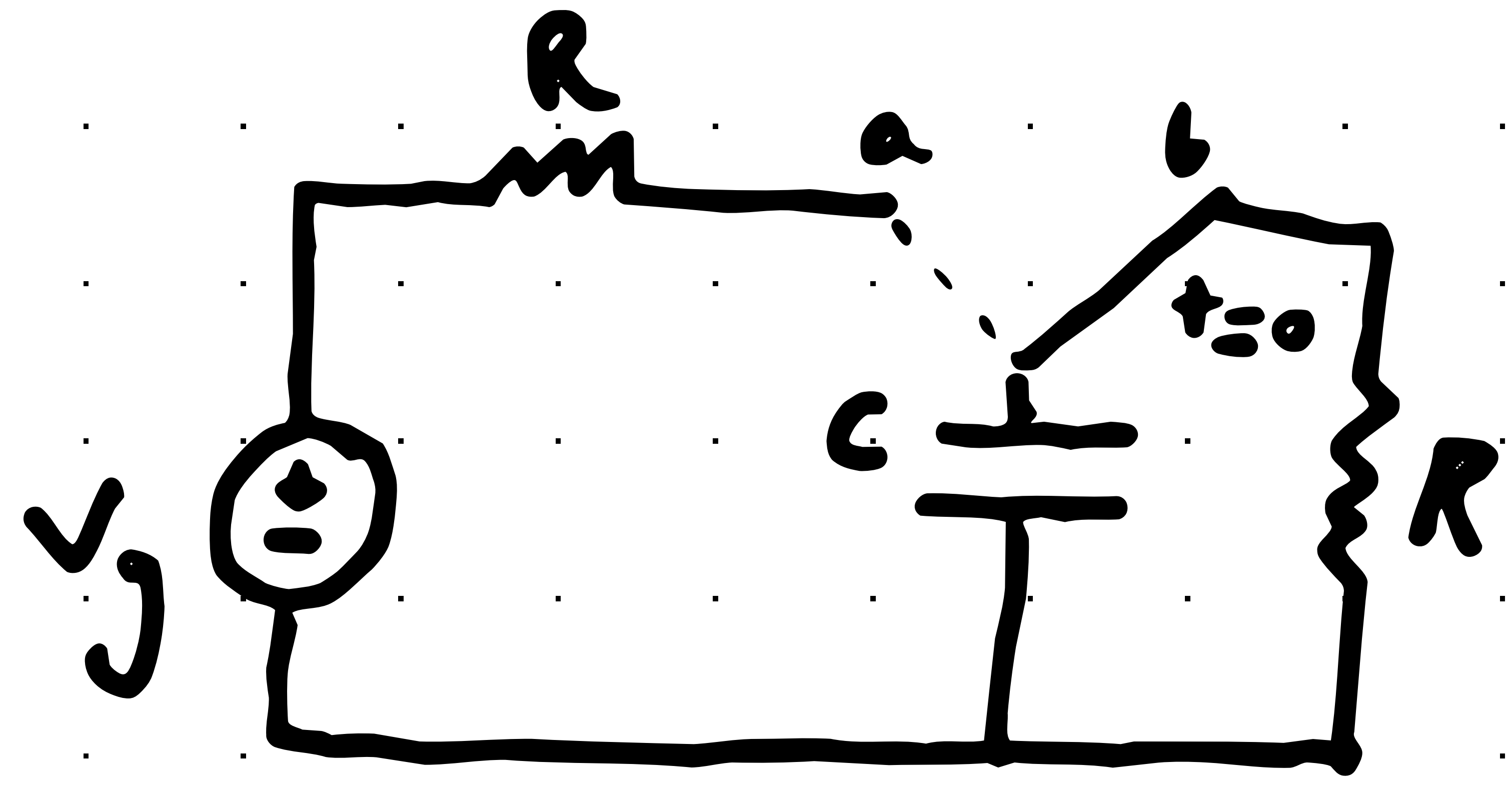
Lecture 18

ECED 2□□□

The Natural Response of an RC Circuit

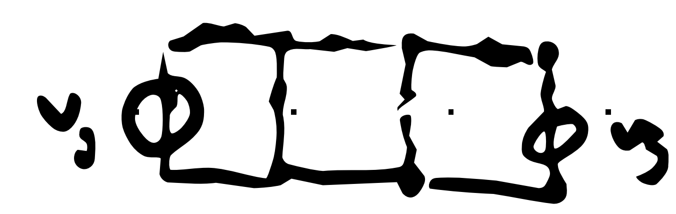
- Forced Response means source after closing the switch
- Natural Response means no source after closing the switch.



At $t=0$, the switch is moved to position B.

Lab Test checklist

1) • Create circuit



• Penalized for blowing fuse

• (parallel vs series for voltage)

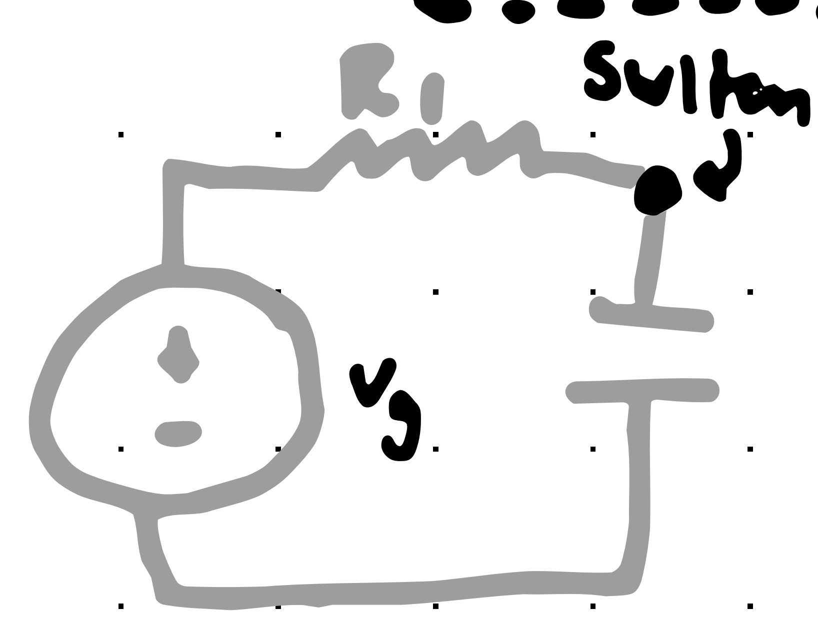
• Can sit anywhere

2) and 3)

Based on lab questions.

- R Thev for example
- Superposition
- Op Amps
- Oscilloscope
- RC and RL

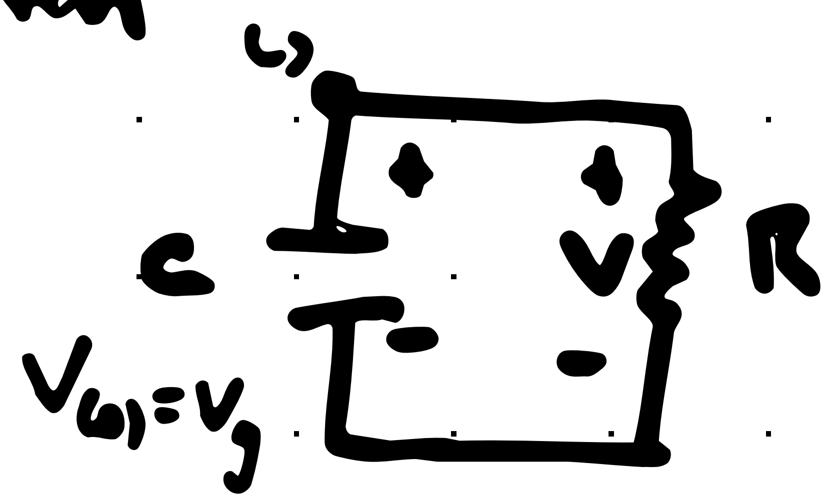
Before moving the switch



At $t=0$, the switch moved.

So the initial charge for the capacitor is V_g

After moving the switch



$$\frac{V}{R} + C\frac{dv}{dt} = 0$$

Solve the above D.E

$$V(t) = V(0)\, e^{-\frac{t}{RC}}$$

The voltage across the capacitor is not changing instantaneously, so $V_c(0^+) = V_c(0^-) = V_c(0)$

The current is changing across the

0_+ = Immediately after moving switch

0_- = Immediately before moving switch

Assume

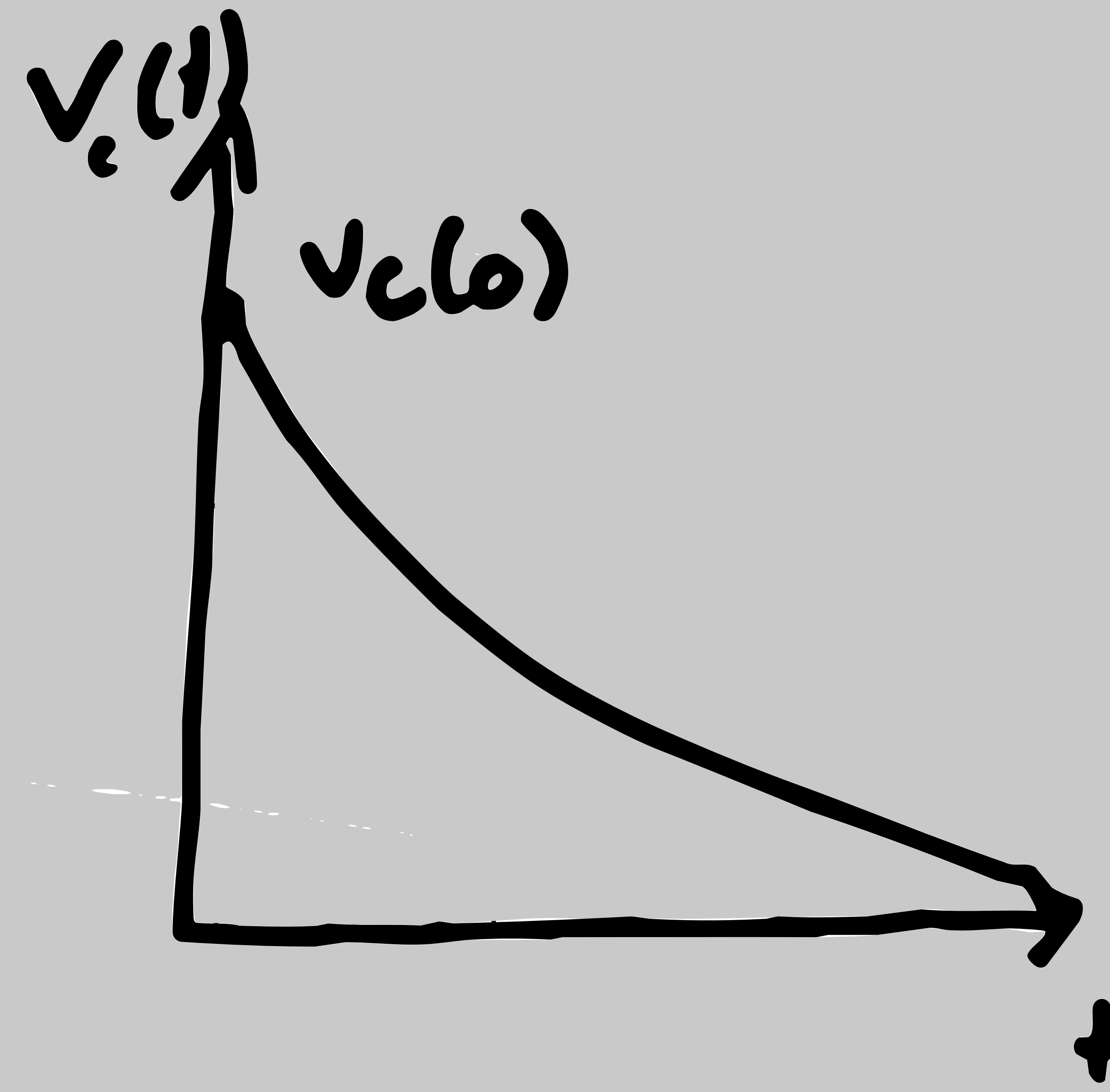
$\tau = RC$ (time constant

$$V_c(t) = V_c(0)\, e^{-\frac{t}{\tau}}$$

$$i(t) = \frac{v(t)}{R} = \frac{v(0)}{R} e^{-\frac{t}{\tau}}$$

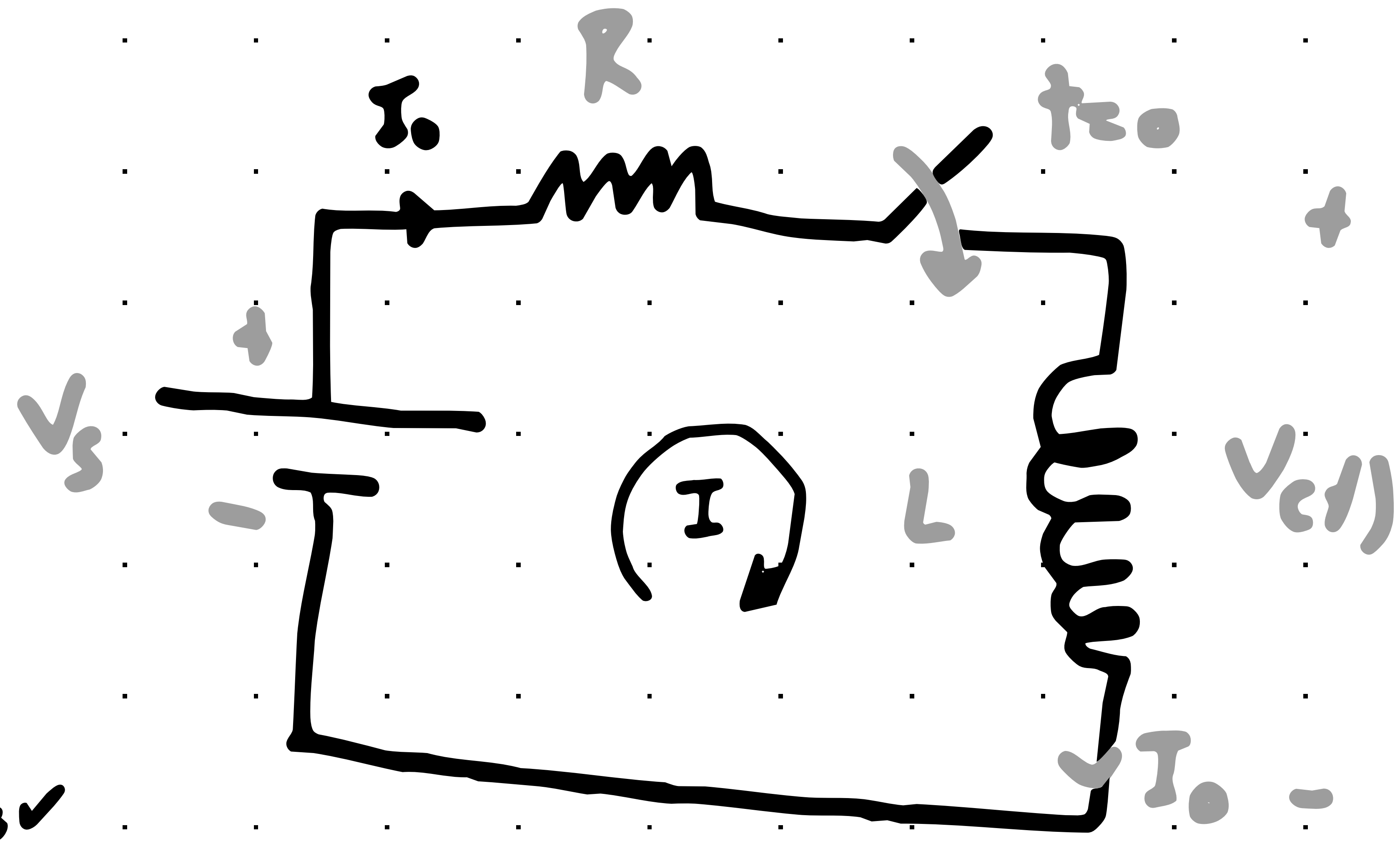
$$P_c(t) = \frac{V_c(0)^2}{R}\, e^{-\frac{2t}{\tau}}$$

$$W_c(t) = \int_0^t P_c(t)\,dt = \frac{1}{2} C \left(V_c(0)\right)^2 \left(1 - e^{-\frac{2t}{\tau}}\right)$$



Step Response of RL

(General)



$$\sum_{\text{loop}} V = 0$$

$$V_s = IR + L\frac{di}{dt}$$ ← First ORder DE

$$\frac{di}{dt} = \frac{-Ri + V_s}{L} = -\frac{R}{L}\left(i - \frac{V_s}{R}\right)$$

$$\int_{I_0}^{I_0} \frac{di}{\left(i - \frac{V_s}{R}\right)} = -\int_{t_0}^{t} \frac{R}{L}\, dt \rightarrow \ln\left(i - \frac{V_s}{R}\right)\Big|_{I_0}^{I(t)} = -\frac{R}{L}t\Big|_{t_0}^{t}$$

$$\left(\frac{\left(i(t) - \frac{V_s}{R}\right)}{I_0 - V_s/R}\right) = e^{-\frac{R}{L}(t - t_0)}$$

For simplicity, if $t_0 = 0$ then $= e^{-\frac{R}{L}t}$

$$I(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}t}$$

General EQn for Step and natural.

Natural Response

* when no source, these equal zero, so happens to be the natural Response! Eqn

$$V(t) = L\frac{di}{dt} = L\left(-\frac{R}{L}\right)\left(I_0 - \frac{V_s}{R}\right)e^{-\frac{R}{L}t}$$

$$= (V_s - I_0 R)\, e^{\left(-\frac{R}{L}\right)t}$$

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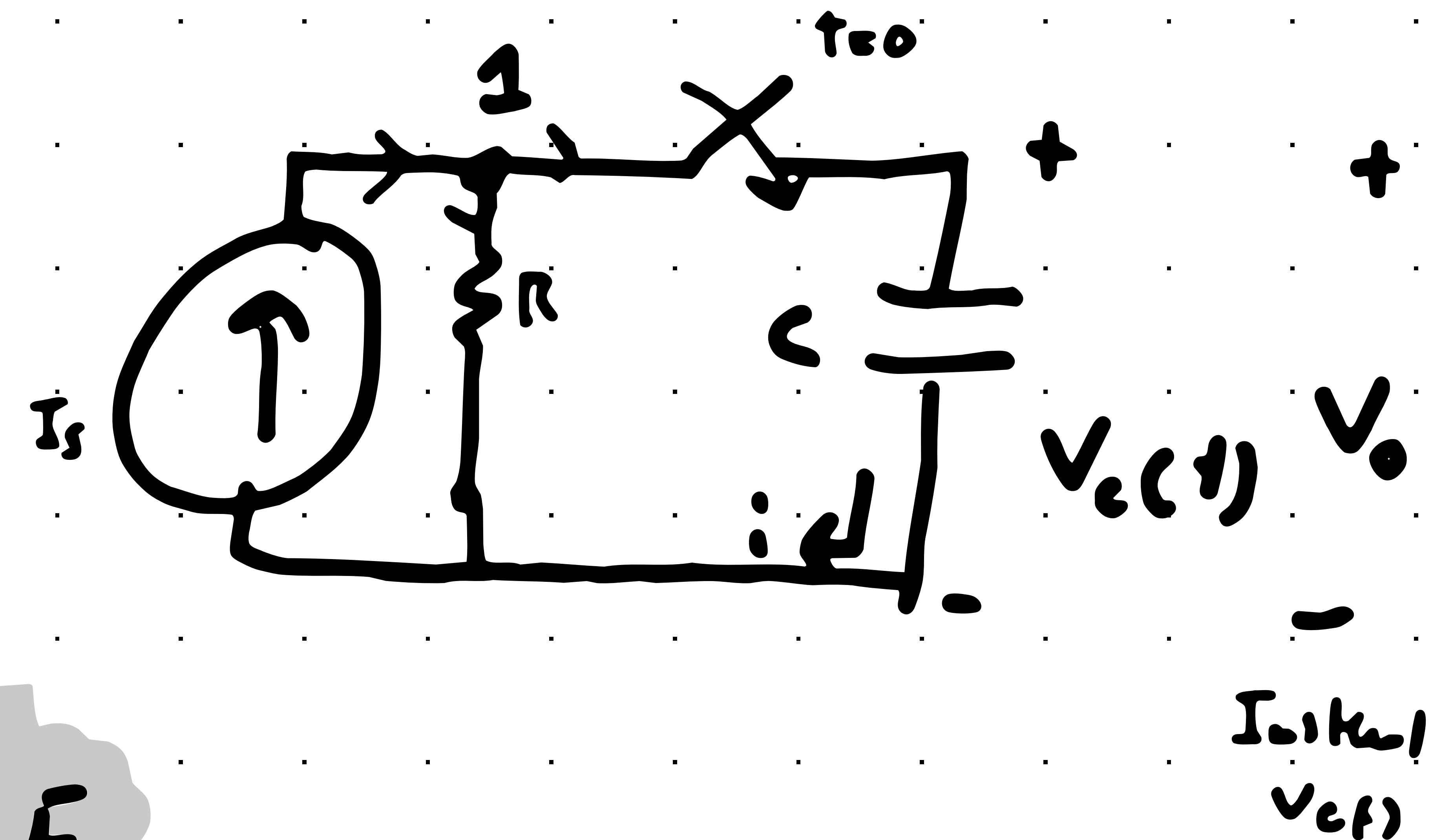
b) Step Response of RC Circuit



$\sum I = 0$

Node 1

$$I_s = \frac{v_o}{R} + C\frac{dv_c}{dt}$$ D.E

$$\frac{dv_c}{dt} + \frac{v_c}{RC} = \frac{I_c}{C}$$

$$V_c(t) = I_sR + (V_0 - I_sR)e^{-t/RC} \qquad t \geq 0$$

$$i(t) = \left(I_s - \frac{V_o}{R}\right)e^{-\frac{t}{RC}} \qquad t \geq 0^+$$

Now, we only ever derived these equations for one resistor, and inductor! What I'll do in tomorrow's notes, is show how to create an equivalent system....