Lecture 19 ECED 2☐☐☐

For RL Step Response

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}t}$$

Need Voltage Source

I_0 = current in inductor at $t=0$

For RC Step Response

$$V_c(t) = I_s R + (V_0 - I_s R) e^{-\frac{t}{RC}}$$

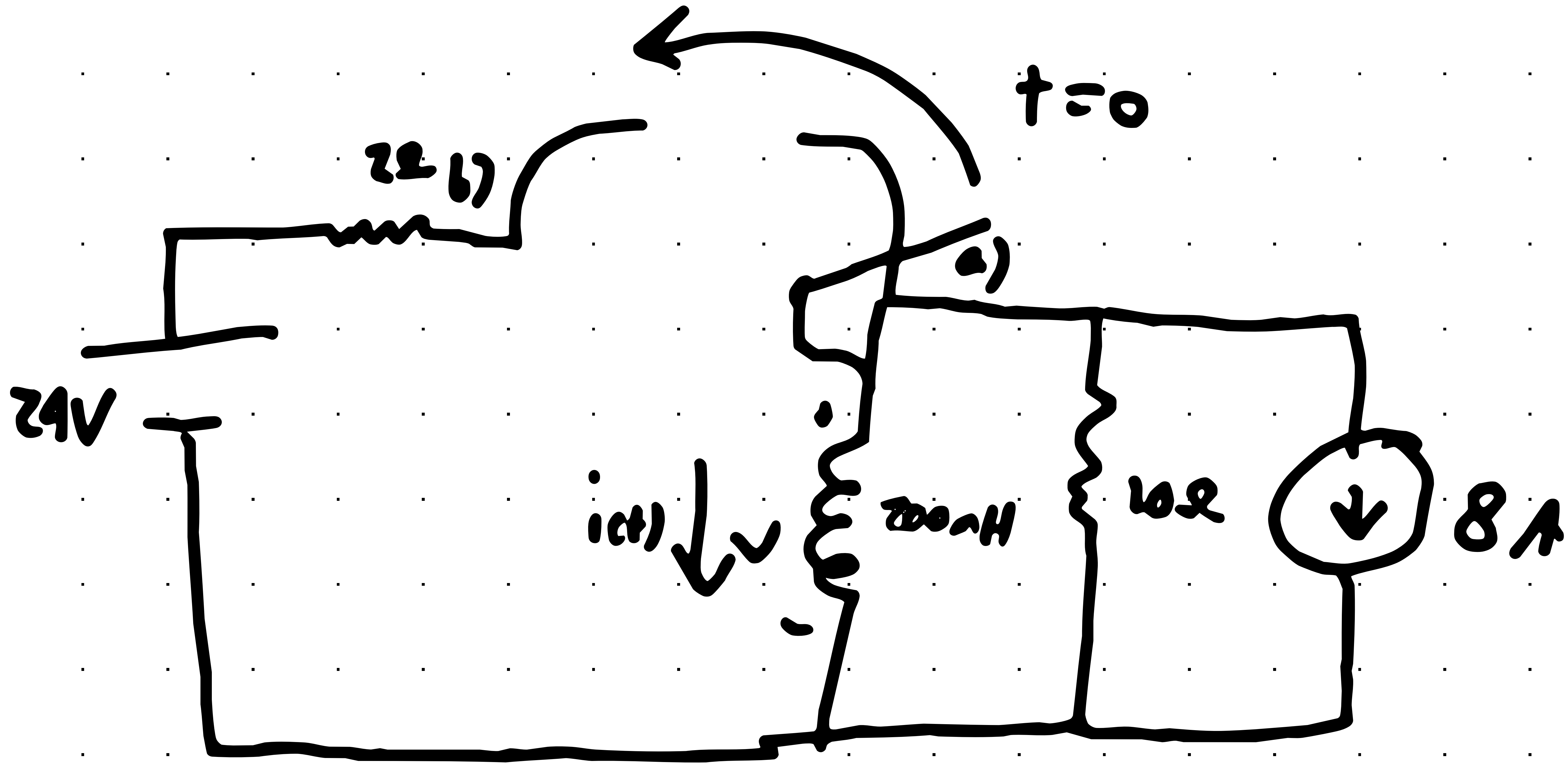
Need Current Source

Example

(a) Find an Expression for $i(t)$ for $t \geq 0$

(b) What is the initial voltage across the inductor just after the Switch has been moved to Position B?

(c) How many milliseconds after the Switch has been moved does the inductor voltage equal 24V?

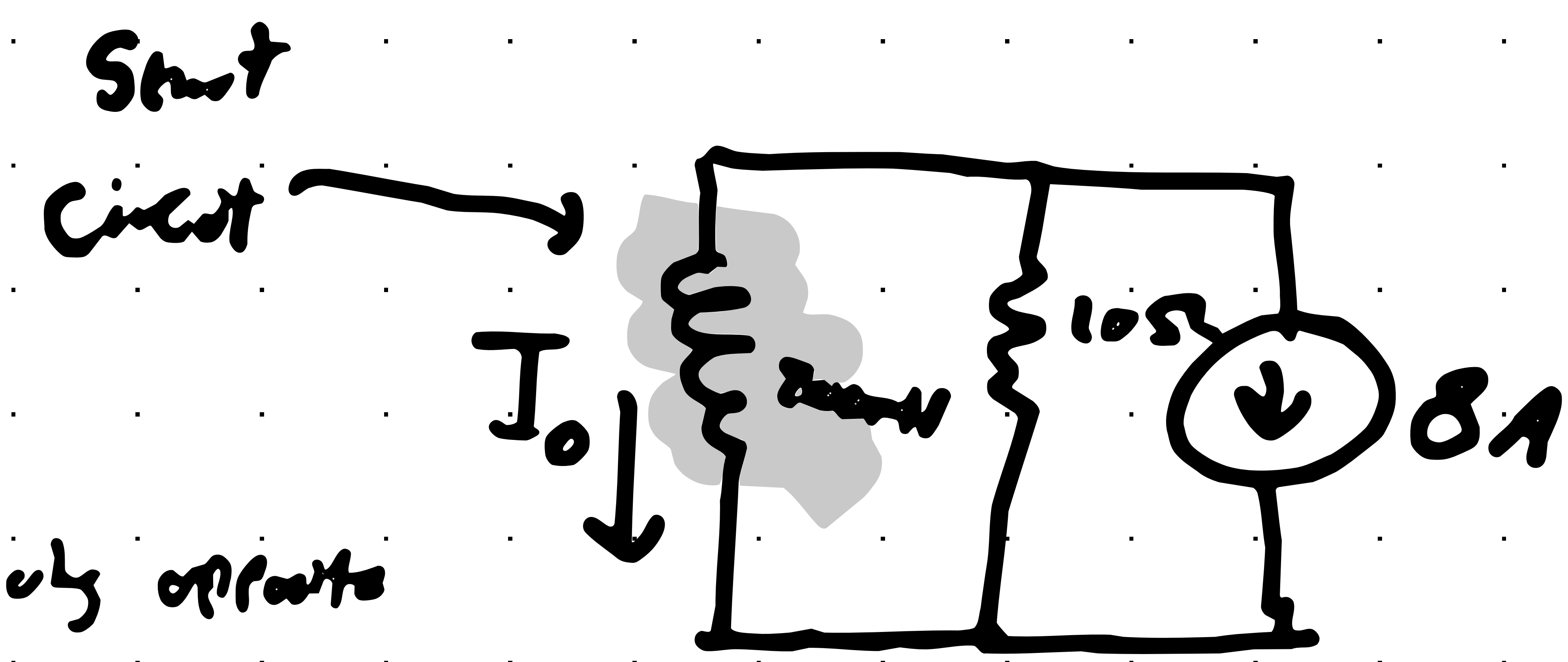


Solution

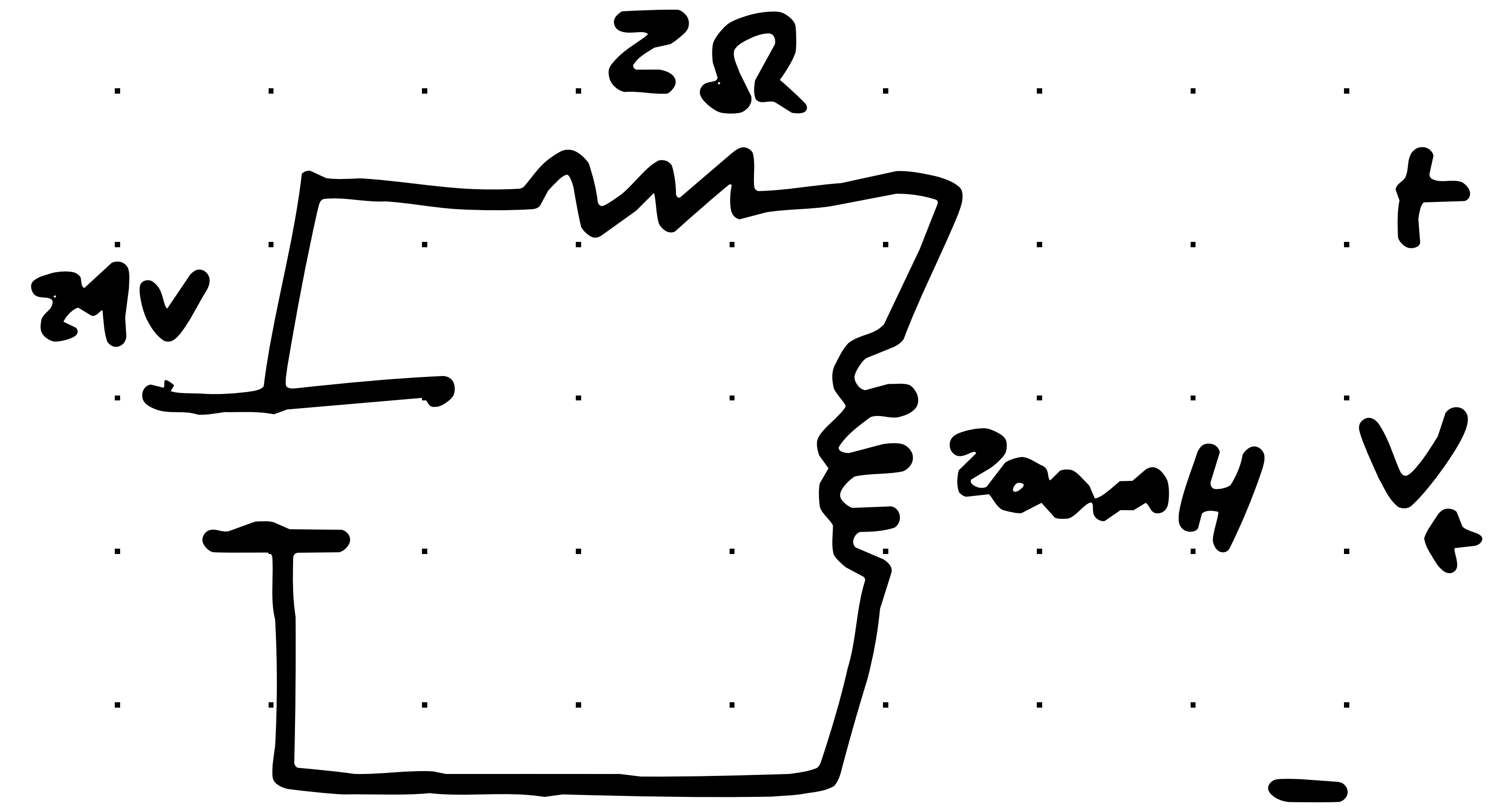
Before moving the switch

$I_0 = -8A$

(current goes opposite to source)



After moving the switch



$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}t}$$

One resistor ✓

Voltage Source ✓

$$I_{(t)} = \frac{24}{2} + \left(-8 - \frac{24}{2}\right) e^{-\frac{2}{200\times10^{-3}} t}$$

(a) $i(t) = 12 - 20e^{-10t}$ $t \geq 0$

(b) $V(t) = L\frac{di}{dt} = 200\times10^{-3}\left(0 - 20e^{-10t}(-10)\right)$

$V(t) = 40e^{-10t}$ $t \geq 0^+$

Initial voltage at $T=0$ is $40V$ $(40e^{-10(0)})$

(c)

$24 = 40e^{-10t}$

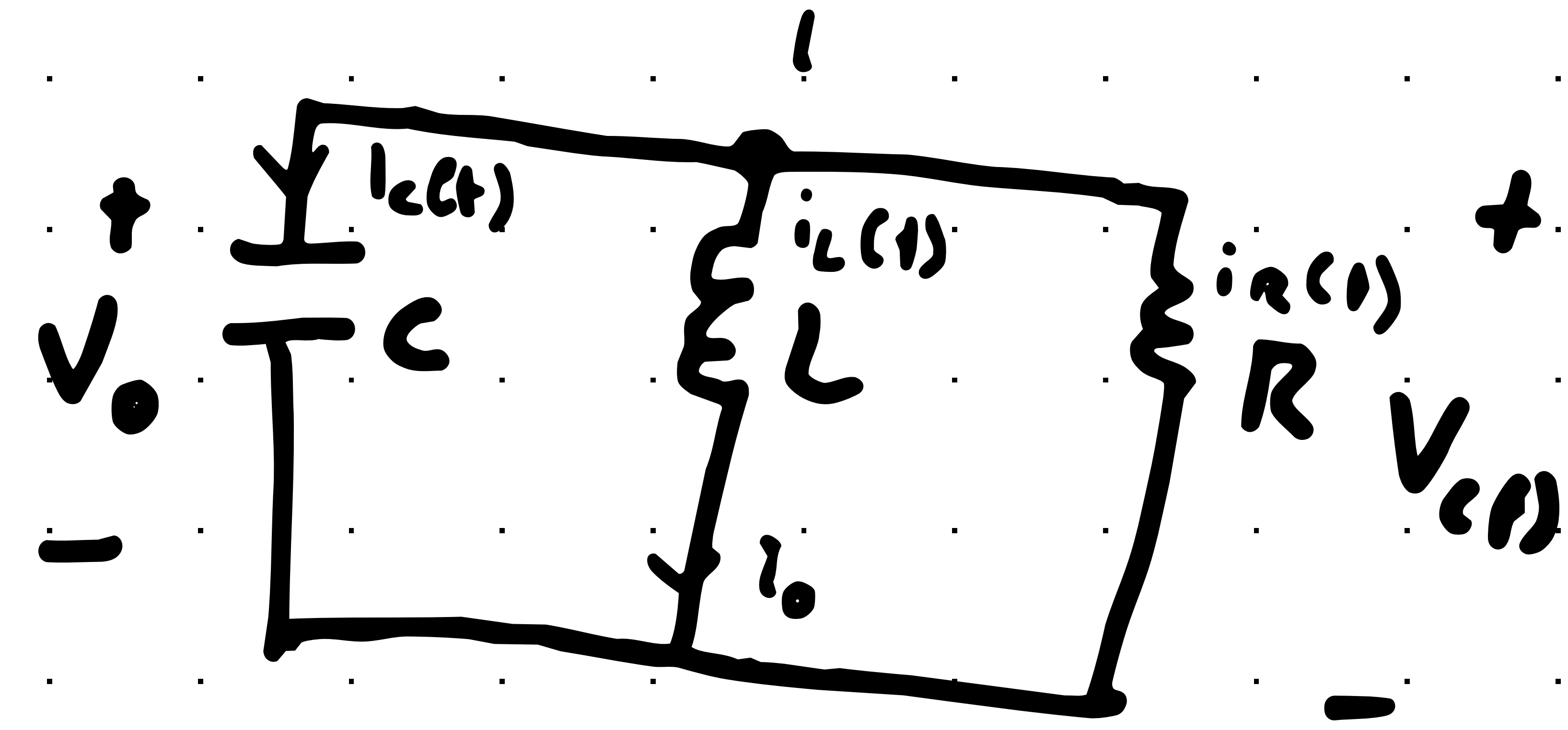
$\ln(24) = \ln(40) + \ln e^{-10t}$

$\ln 24 - \ln 40 = -10t$ $t = 51.08ms$

Introduction to the Natural Response of a Parallel RLC Circuit

$$\sum_{\text{Node 1}} I = 0$$

(Small i is i(t), Not writing to simplicity.)



$$i_C + i_R + i_L = 0$$

$$C\frac{dv}{dt} + \frac{V}{R} + \frac{1}{L}\int v\,dt = 0$$

Ugly DE

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}(v) = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}V = 0$$

Assume $V(t) = Ae^{st}$

$$\frac{dv}{dt} = Ase^{st}$$

$$\frac{d^2v}{dt^2} = As^2e^{st}$$

$$As^2e^{st} + \frac{As}{RC}e^{st} + \frac{A}{LC}e^{st} = 0$$

$$As^2e^{st} + \frac{As}{RC}e^{st} + \frac{A}{LC}e^{st} = 0$$

$$\left(S^2 + \frac{1}{RC}S + \frac{1}{LC}\right)Ae^{st} = 0$$

$$S^2 + \frac{1}{RC}S + \frac{1}{LC} = 0$$

Note:

$$S = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$S = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - 4(1)\left(\frac{1}{LC}\right)}}{2}$$

$$S_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{LC}\right)}$$

$$S_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{LC}\right)}$$

These are known as complex frequencies

$$\alpha = \frac{1}{2RC}$$

Neper frequency

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

Resonant radian frequency

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

Based on α and ω_0 values, we are expecting three different cases.

Case 1 If $\alpha^2 > \omega_0^2$ (Two distinct real solutions)

This case is called (over damped)

Solution $\boxed{v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}}$

For A_1 and A_2 calculations, we use initial conditions.

$$v(0)^+ = A_1 + A_2 \; ;$$

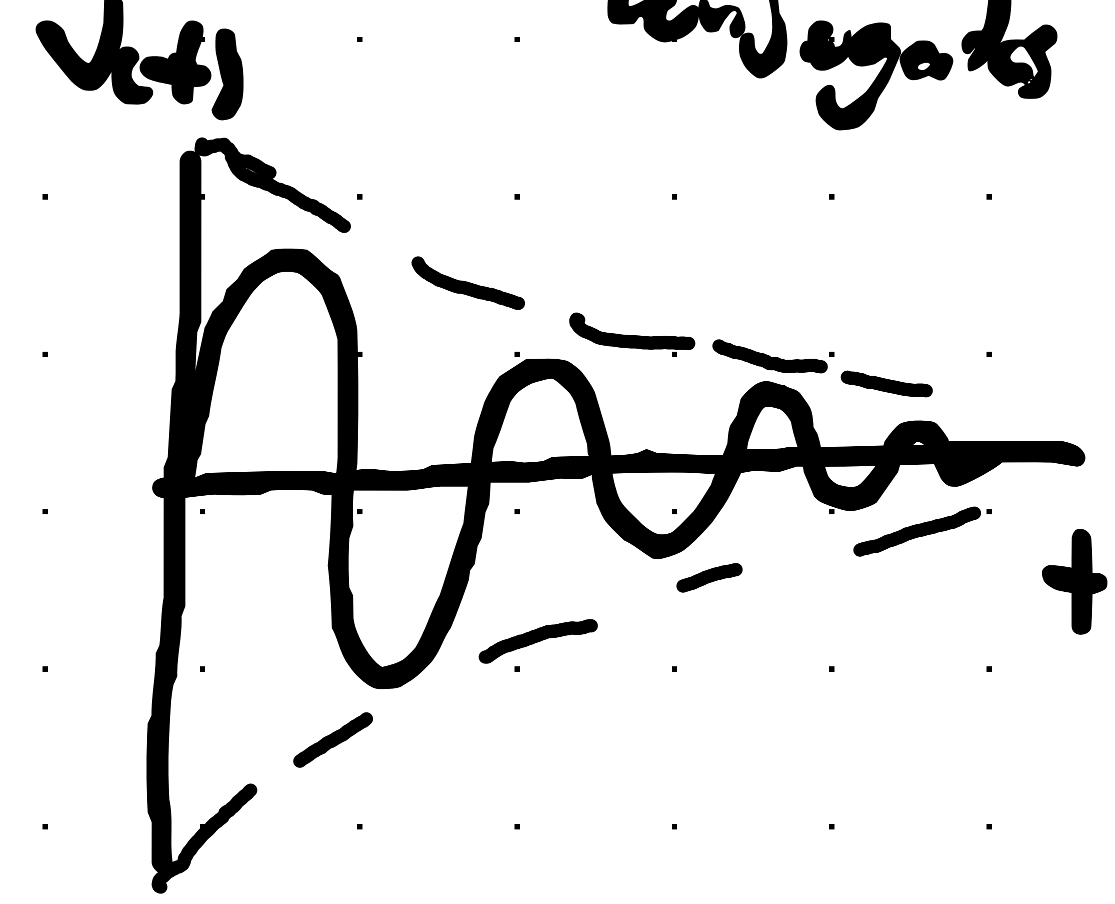
$$\frac{dv(0)^+}{dt} = s_1 A_1 + s_2 A_2 = \frac{i_c(0)}{c}$$

- Step one: Solve for α and ω
- Step two: Compare α and ω
- Step three if α^2 is less than ω, use case 1

Case 2 If $\alpha^2 < \omega_0^2$ (under damped case)
Two complex conjugates

$$V(t) = \beta_1 e^{-\alpha t} \cos\omega_d t + \beta e^{-\alpha t} \sin\omega_d t$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$



For finding β_1 and β_2, use initial conditions

$$V(0)^+ = \beta_1$$

$$\frac{dv^+(0)}{dt} = -\alpha\beta_1 + \omega_d \beta_2$$

Case 3 if $\omega^2 = \alpha^2$ (Critically damped) (Repeated Roots) (Real)

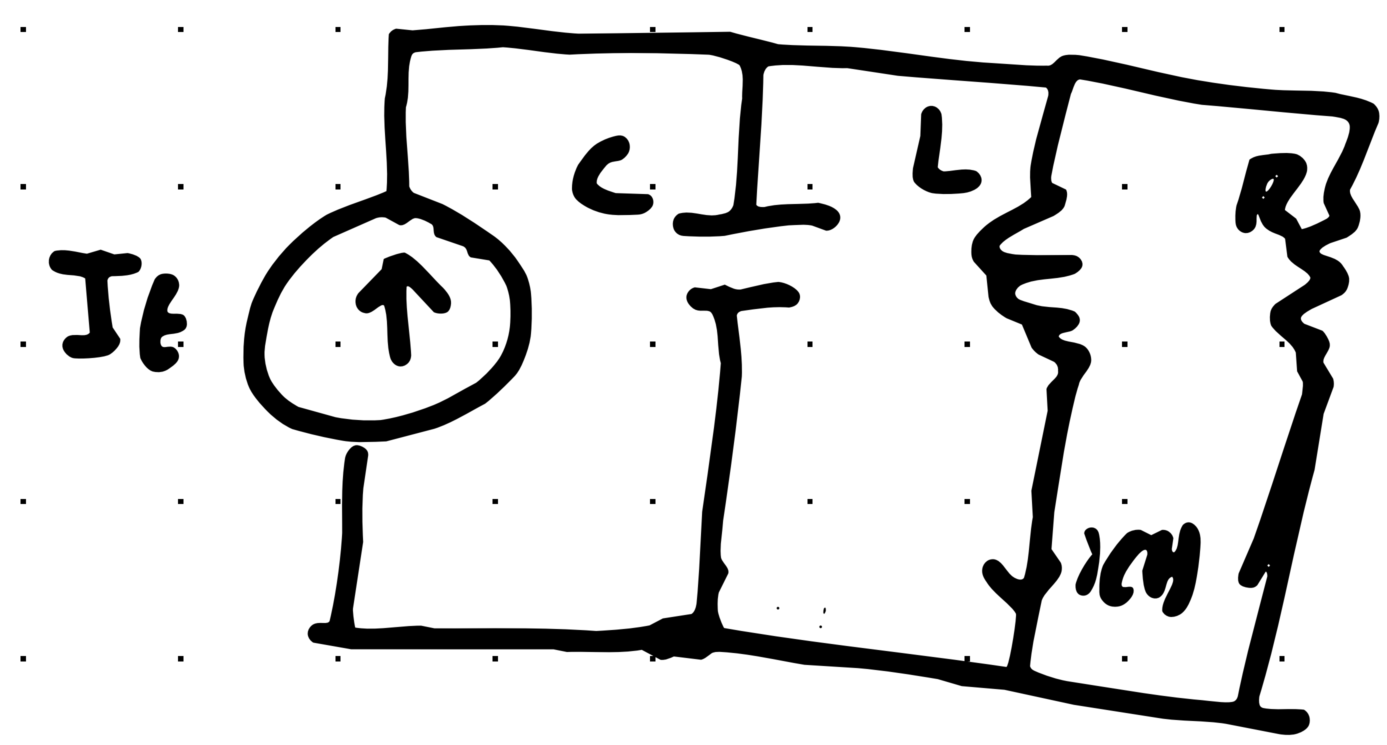
$$V(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

→ for D_1 and D_2, use Initial conditions

$$V_0(t) = D_2$$

$$\frac{dV(0)^+}{dt} = D_1 - \alpha D_1$$

In Case of Step response RLC Parallel



Case ①

If $\alpha^2 > \omega_0^2$, then

$$i_L(t) = I_f + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Case ②

If $\alpha^2 < \omega_0^2$, then

$$i_L(t) = I_f + B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t$$

Case ③

If $\alpha^2 = \omega_0^2$, then

$$i(t) = I_f + D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

$$\alpha = \frac{1}{2RC} \qquad \omega_0 = \sqrt{\frac{1}{LC}}$$