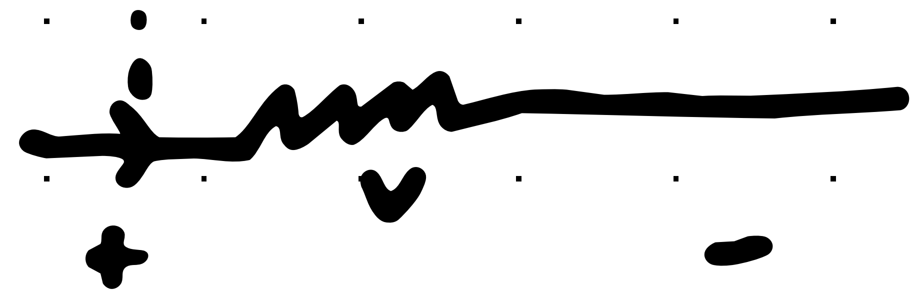
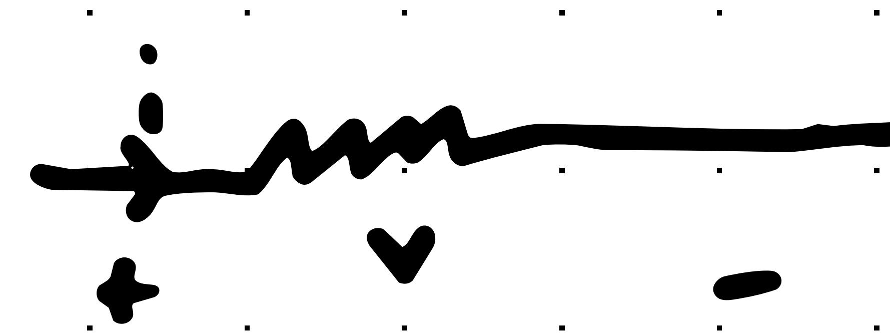
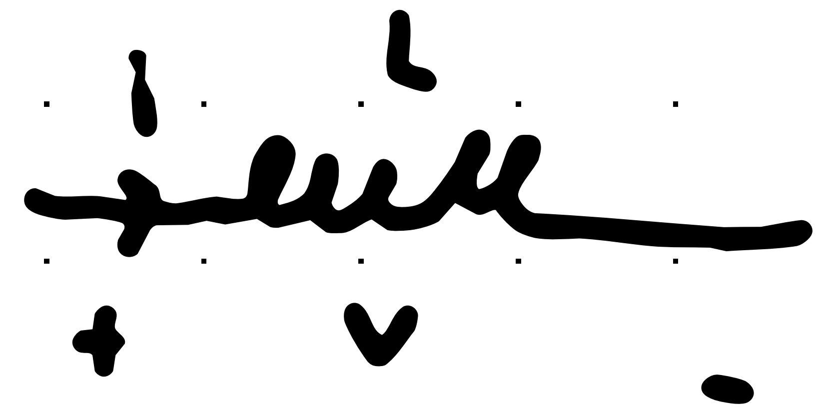
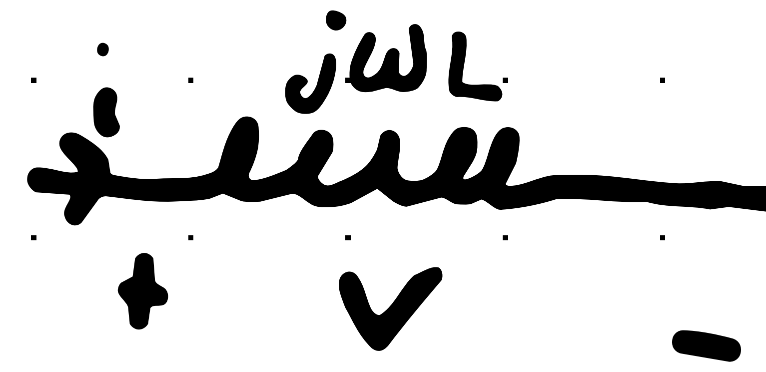
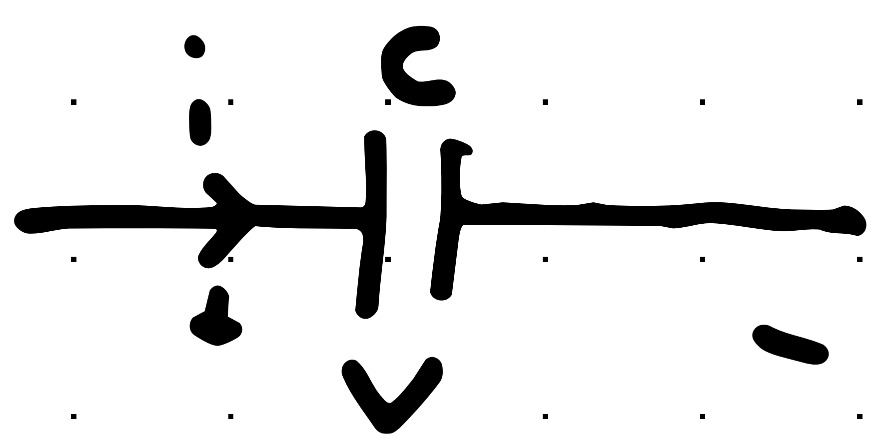
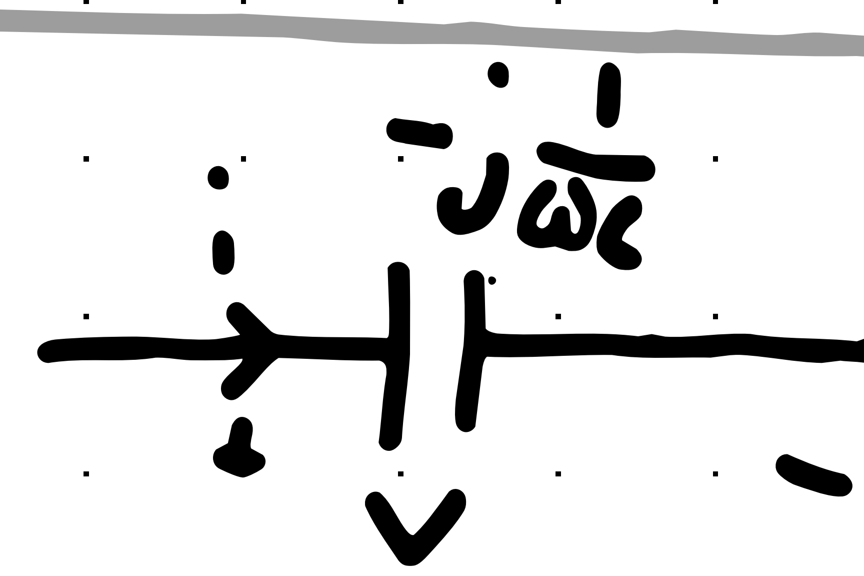
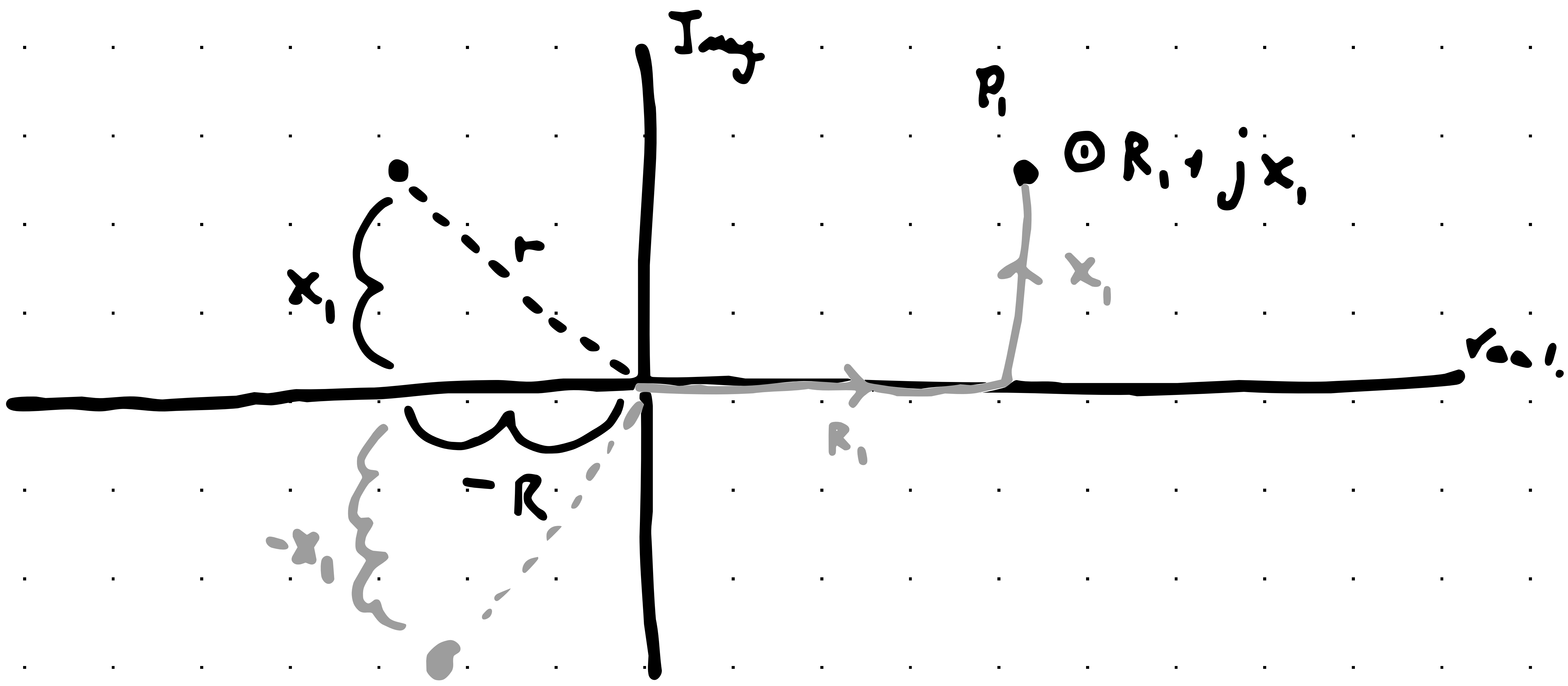
Lecture 20 ECED 2□□□

Sinusodial Steady State

Element	Time domain	Phasor domain
① Resistor	 $V = IR$	 $V = IR$
② Inductor	 $V = L\frac{di}{dt}$	 $V = i(j\omega L)$
③ Capacitor	 $V = \frac{1}{C}\int i\,dt + V(t_0)$	 $V = i\left(-j\frac{1}{\omega C}\right)$

Complex Numbers



Complex plane

P1 is represented by either:

Rectangular Form $R_1 + jX_1$

Polar Form $r\angle\theta$

$$r\angle\theta = R_1 + jX_1 = r\cos\theta + jr\sin\theta,$$

$$R_1 jX_1 = \sqrt{R_1^2 + X_1^2}\ \angle \tan^{-1}\left(\frac{X}{R}\right) = r\angle\theta$$

For the Second Quadrant

$$-R_1 + jX_1 = \sqrt{R_1^2 + X_1^2}\ \angle 180 - \tan^{-1}\frac{X_1}{R_1}$$

For the Third Quadrant

$$-R_1 - jX_1 = \sqrt{R_1^2 + X_1^2} \angle 180 + \tan^{-1}\frac{X_1}{R_1}$$

For the fourth Quadrant

$$-R_1 - jX_1 = \sqrt{R_1^2 + X_1^2} \angle -\tan^{-1}\frac{X_1}{R_1}$$

As Well $j = \sqrt{-1}$, $j^2 = 1$

Adding or Subtracting two Complex Numbers

$$\Rightarrow (R_1 + jX_1) \pm (R_2 + jX_2) =$$

$$= (R_1 \pm R_2) + j(X_1 \pm X_2)$$

$$\Rightarrow r_1 \angle \theta + r_2 \angle \theta_2 =$$

$$= r_1 \cos\theta_1 + j r_1 \sin\theta_1 + r_2 \cos\theta_2 + j r_2 \sin\theta_2 =$$

$$= (r_1 \cos\theta_1 + r_2 \cos\theta) + j(r_1 \sin\theta_1 + r_2 \sin\theta_2)$$

Multiplying or dividing two complex numbers

$$(R_1 + jX_1)(R_2 + jX_2) = R_1R_2 + jR_1X_2 + jR_2X_1 - X_1X_2$$

$$= (R_1R_2 - X_1X_2) + j(R_1X_2 + R_2X_1)$$

Multiplication

$$\left(\frac{R_1 + jX_1}{R_2 + jX_2}\right)\left(\frac{R_2 - jX_2}{R_2 - jX_2}\right)$$

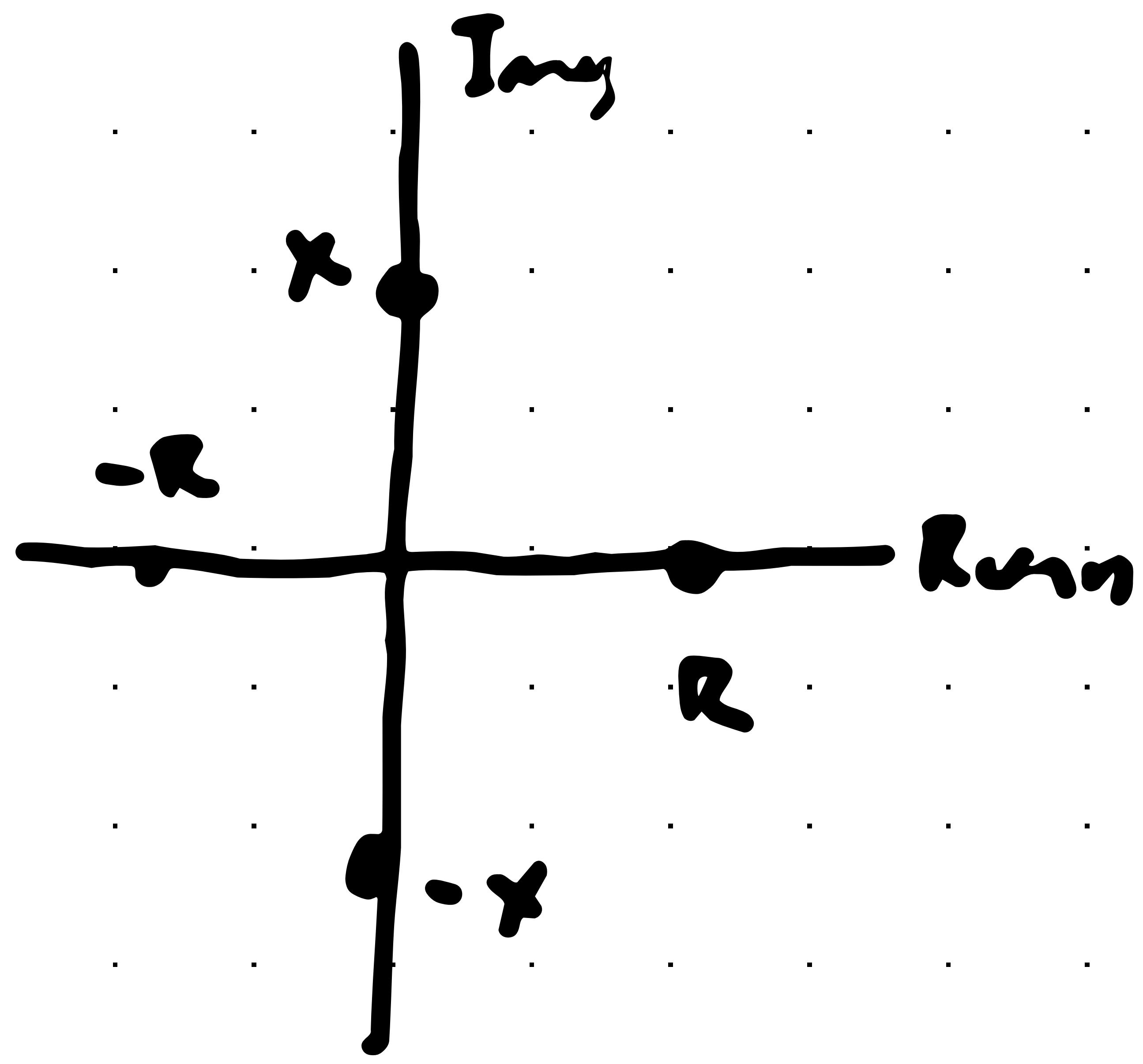
complex conjugate needed

Division

Polar

$$\frac{r_1\angle\theta_1}{r_2\angle\theta_2} = \frac{r_1}{r_2}\angle\theta_1 - \theta_2 ,$$

$$(r_1\angle\theta_1)(r_2\angle\theta_2) = r_1r_2\angle\theta_1 + \theta_2$$



$R = R + j0 = R\angle 0$

(All real, so angle is zero)

$jX = X\angle 90^\circ$

(All Imaginary, so angle is 90°)

$-R = R\angle 180^\circ$

$-jX = X\angle 270^\circ$

Resistor

$V = IR$

Angle of Voltage, and Angle of current is the **Same**

No phase Shift.

$5_i = 5\angle 90$

Inductor

$jX_L = jWL = j2\pi fL$

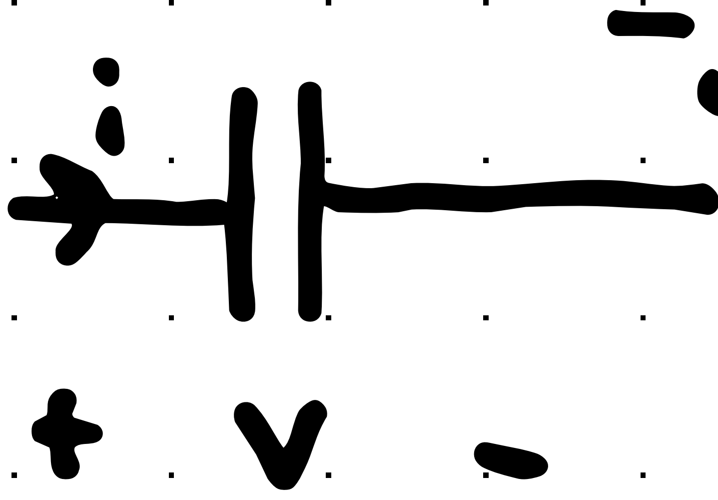
☆ **Reactance** (Ω)

$V = i(jWL)$

$= iWL\angle 90°$

Voltage leads current by 90°. phase Shift is 90°.

Capacitor



$-j\frac{1}{jWC}$

$-j\left(\frac{1}{WC}\right) = -j\frac{1}{2\pi fC}$

$V = i\left(-j\frac{1}{jWC}\right) = i(-jX_c)$

Voltage in phasor domain

- $V_{(t)} = V_m \cos(\omega t + \phi)$ ← Phi

Time Domain

⇓

Phasor

$V = V_m \angle \phi$

Current in phasor domain

- $i_{(t)} = I_m \cos(\omega t + \theta)$

Time Domain

⇓

Phasor

$i(t) = I_m \angle \theta$