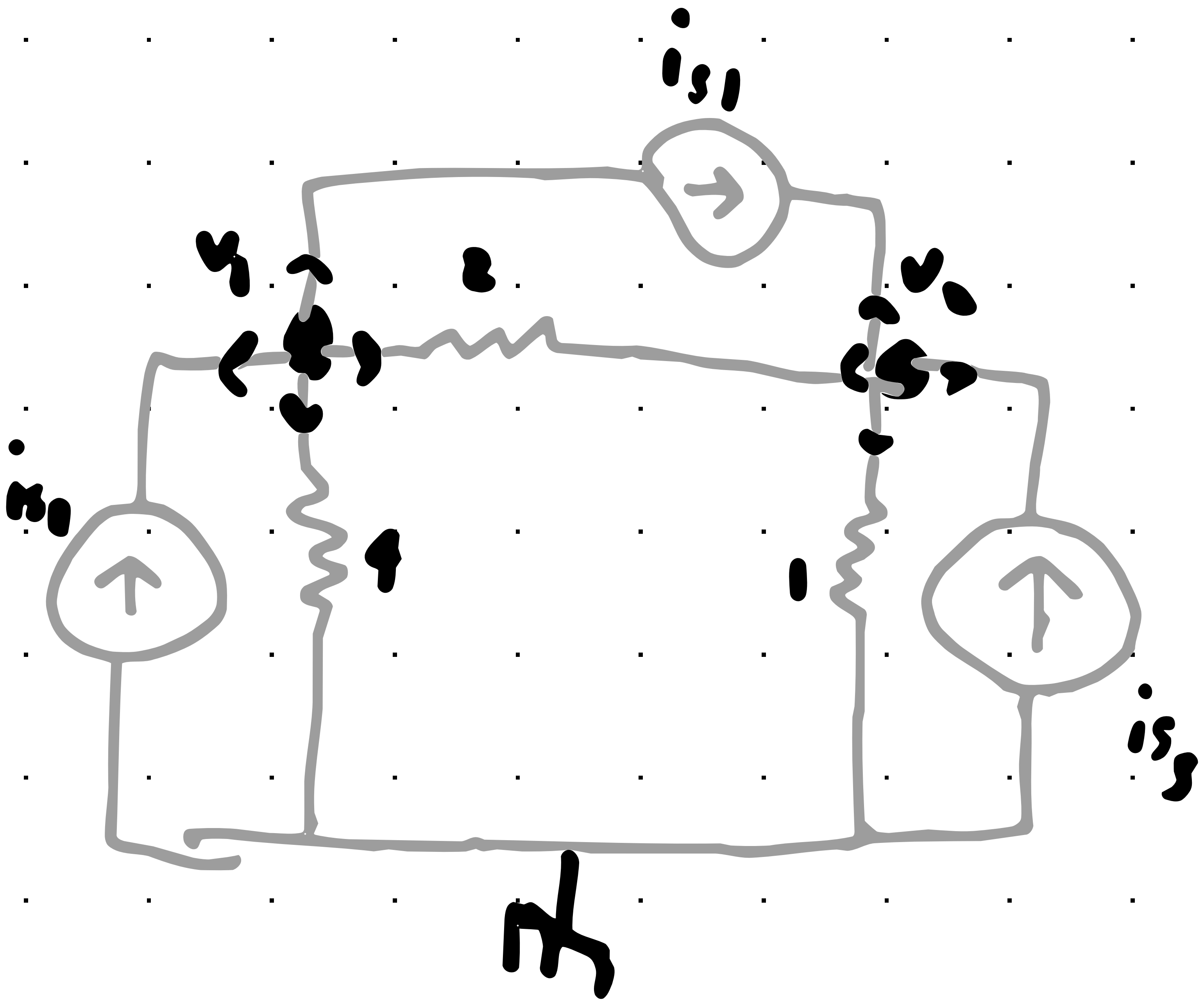
Lecture 7 ECED 2□□□

Nodal Analysis

For the following circuit, write the Node-Voltage equations.



Solution

$$\sum_{\text{Node}_1} I = 0 = -I_{s_2} + I_{s_1} + \frac{V_1}{4} + \frac{V_1 - V_2}{2} = 0$$

$$\sum_{\text{node}_2} = -I_{s_1} - I_{s_3} + \frac{V_1}{1} + \frac{V_2 - V_1}{2} = 0$$

$$\left(\frac{1}{4} + \frac{1}{2}\right)V_1 + \left(-\frac{1}{2}\right)V_2 = i_{s_2} - i_{s_1} \quad (1)$$

$$\left(-\frac{1}{2}\right)V_1 + \left(\frac{1}{1} + \frac{1}{2}\right)V_2 = i_{s_1} + i_{s_3} \quad (2)$$

$$\left(\frac{1}{4}+\frac{1}{2}\right)V_1 + \left(-\frac{1}{2}\right)V_2 = i_{s_2} - i_{s_1} \quad (1)$$

$$\left(-\frac{1}{2}\right)V_1 + \left(\frac{1}{1}+\frac{1}{2}\right)V_2 = i_{s_1} + i_{s_3} \quad (2)$$

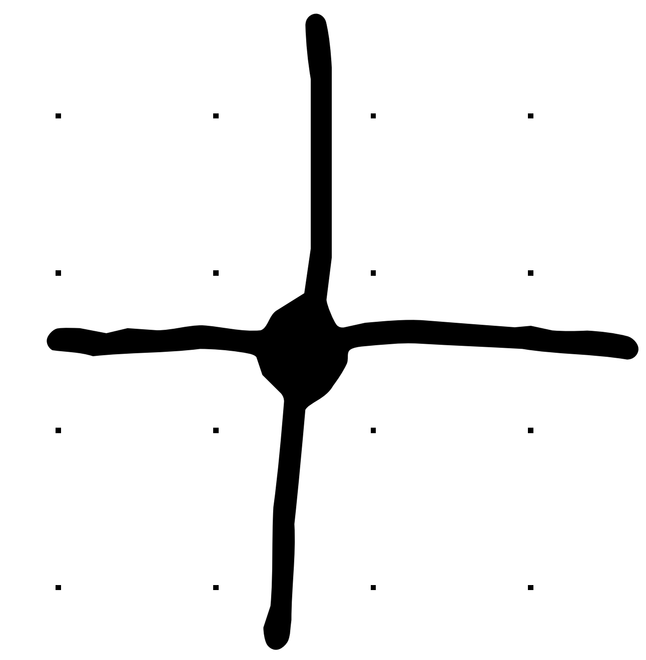
$$\underbrace{\begin{bmatrix} \frac{1}{4}+\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{1}+\frac{1}{2} \end{bmatrix}}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} i_{s_2} - i_{s_1} \\ i_{s_1} + i_{s_3} \end{bmatrix}$$

This is known as the conductance matrix!

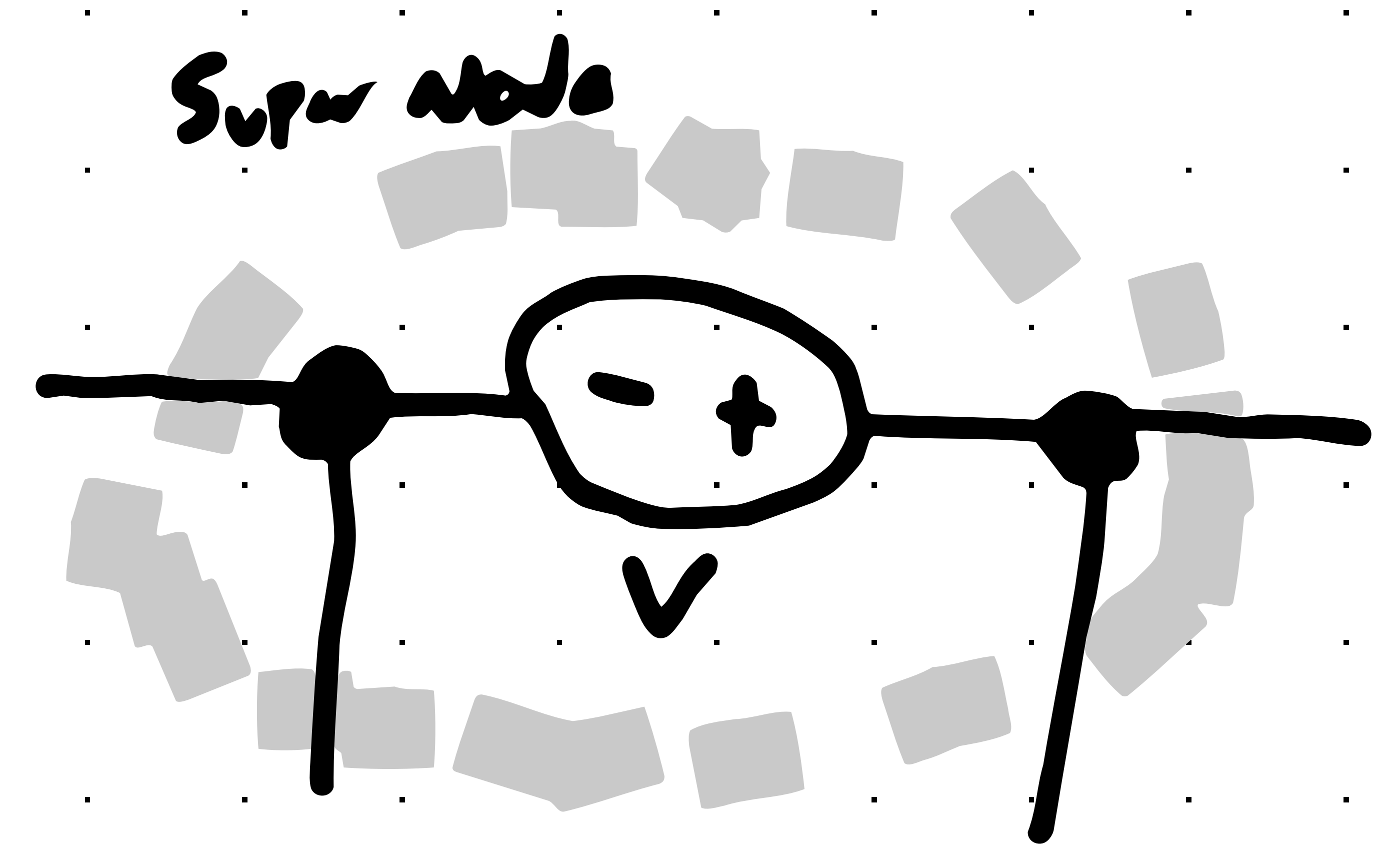
Conductance ($G = \frac{1}{\text{Resistance}}$)

→ Aly doesn't like this method that much, but still wants to discuss it.

Essential Node:



Any node that has three, or more, branches connected together.



Super Node:

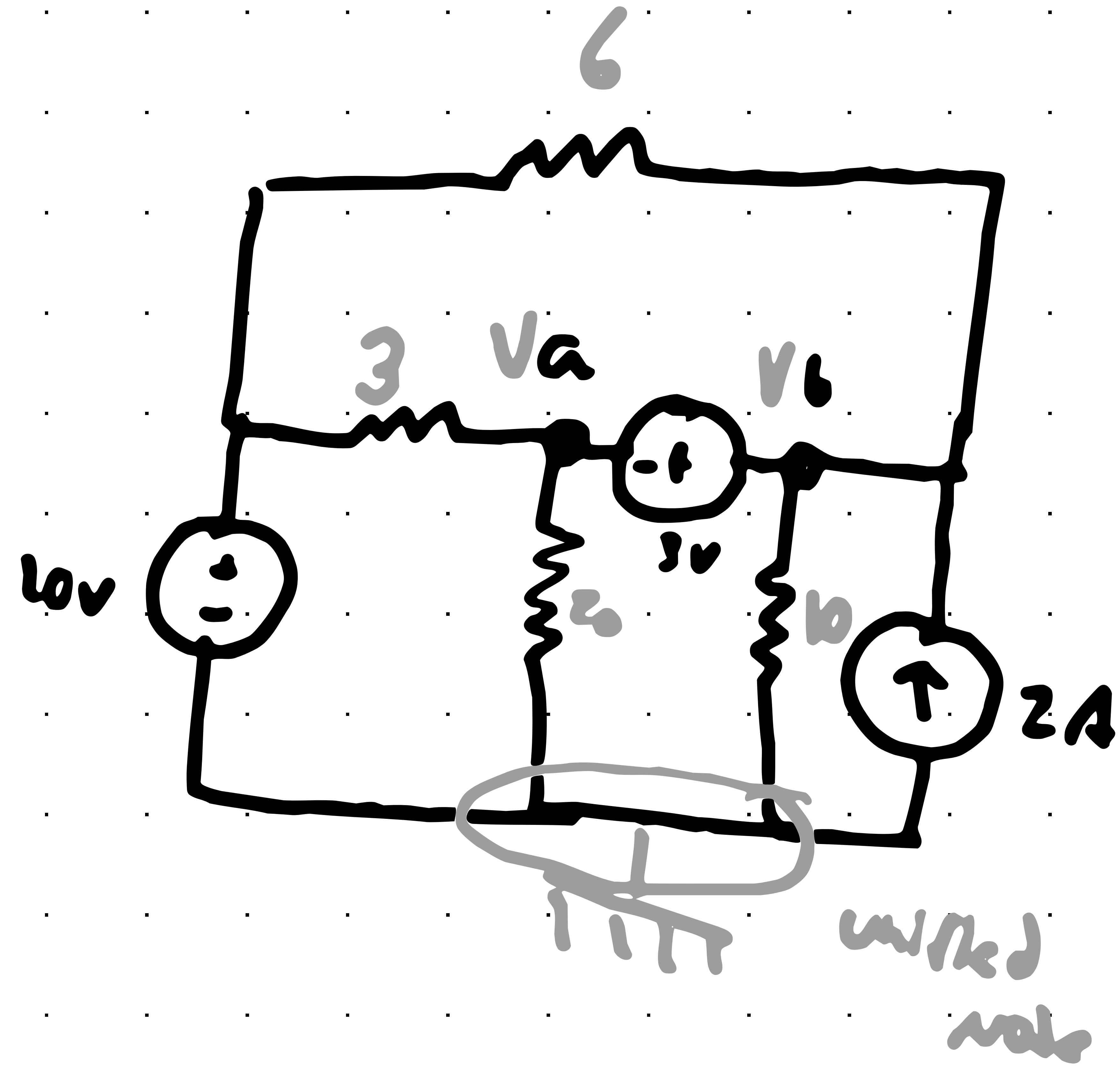
Any two nodes in between a voltage source

There is a proof to say that

$\sum I = 0$ for the entire Supernode.

Example

Use Node Voltage Method to Calculate the Value of V_a



Solution

$$\sum_{\text{Node } a} I = 0 = \frac{V_a - 10}{3} + \frac{V_a}{20} + i_s = 0 \quad (1)$$

$$\sum_{\text{Node } b} I = 0 \quad -i_s + \frac{V_b}{10} + (-2) + \frac{V_b - 10}{6} = 0 \quad (2)$$

Now Substituting Eq. i_s

$$\frac{V_a - 10}{3} + \frac{V_a}{20} + \frac{V_b}{10} + (-2) + \frac{V_b - 10}{6} = 0 \quad (3)$$

$$V_b - V_a = 3 \quad (4)$$

Now Solving these...

$$\frac{V_a - 10}{3} + \frac{V_a}{20} + \frac{V_b}{10} + (-2) + \frac{V_b - 10}{6} = 0 \quad (3)$$

$$V_b - V_a = 3 \quad (4)$$

$$V_a = 10.41V$$

However, there is an easier way to do this....

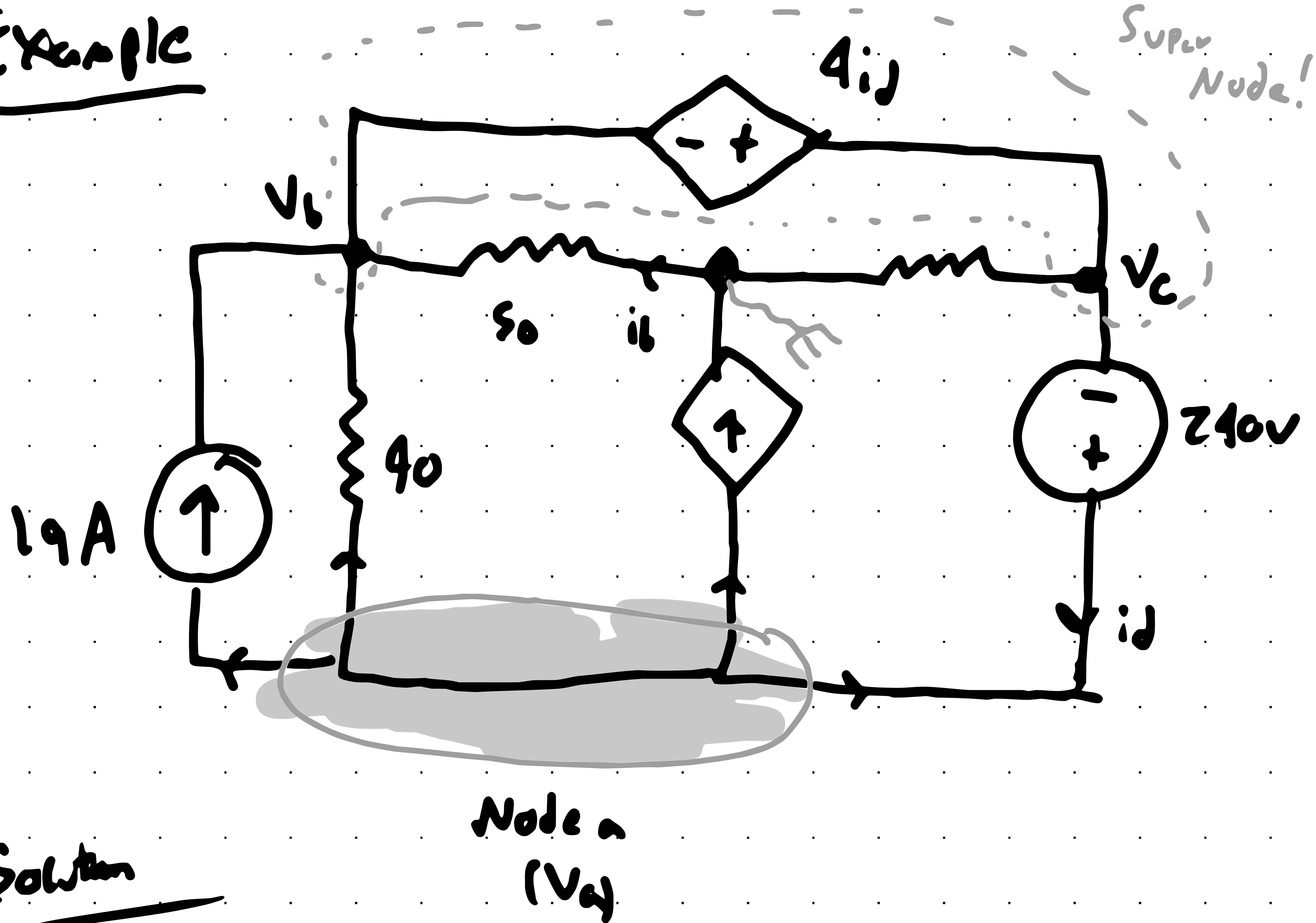
Recognize the Supernode in between V_a and V_b

$\sum I = 0$, Supernode

$$\frac{V_a - 10}{3} + \frac{V_a}{20} + \frac{V_b}{10} + (-2) + \frac{V_b - 10}{6}$$

wow! that's the same equation as the one from both initial equations!

Example



Solution

$\sum I = 0$ @ Node A $\quad 19 + \frac{V_a - V_b}{40} + 2i_b + (-i_d) = 0$

Supernode

$\sum I = 0 \quad -19 + \frac{V_b - V_a}{40} + \frac{V_b - 0}{5} + i_d + \frac{V_c}{6} = 0$

$V_c - V_b = 4i_d$

$\sum I = 0$ node A $\quad i_d = 2i_b + \frac{V_a - V_b}{40} + 19$

$i_b = \frac{0 - V_b}{5}$