Sketching

Exponentials and polynomials

① Small time (region near zero)

Can be written with one taylor series member

$e^{u} = 1 + u$

② Intermediate time

1- Find the critical points = $y'(t) = 0$ $\hookrightarrow t$ ✓ $\hookrightarrow y$ ✓

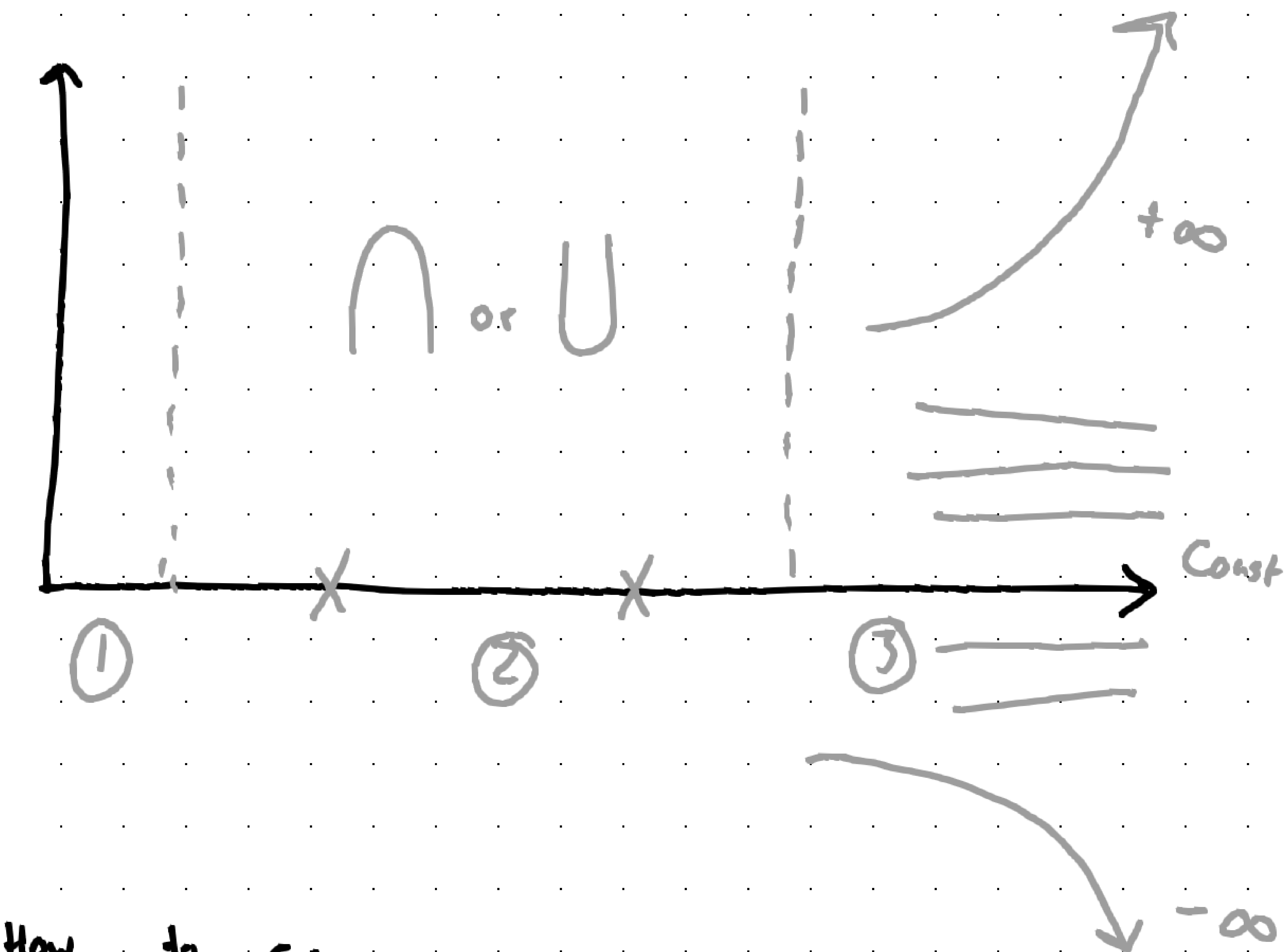
2 - Check whether $y(t)$ crosses the x axis

$y(t) = 0 \longrightarrow t$ ✓

There might not be enough to satisfy 1 and 2 so we might not have a min/max, or we won't cross an axis.

③ Big Time

Check $y(t)$'s end behavior



How to Calculate $t \to \infty$

Example:

$y(t) = te^{-t}$ $\quad t \to \infty \quad \Rightarrow \quad y(t) = \dfrac{t}{e^{-t}} = \dfrac{1}{e^{-t}}$

Goes to zero

$$t \to \infty$$

$y(t) = e^{-\infty} + t \quad \Rightarrow \quad 0 + \infty = \infty$

Goes to infinity

Example Sketch

$$G(t) = G_0 + \underbrace{\frac{K_1^2 G_1}{2}} t^2 e^{-K_1 t} \quad , \quad y(0) = G_0, \quad G_0 > 0$$

Find Peaks!

$$\downarrow$$

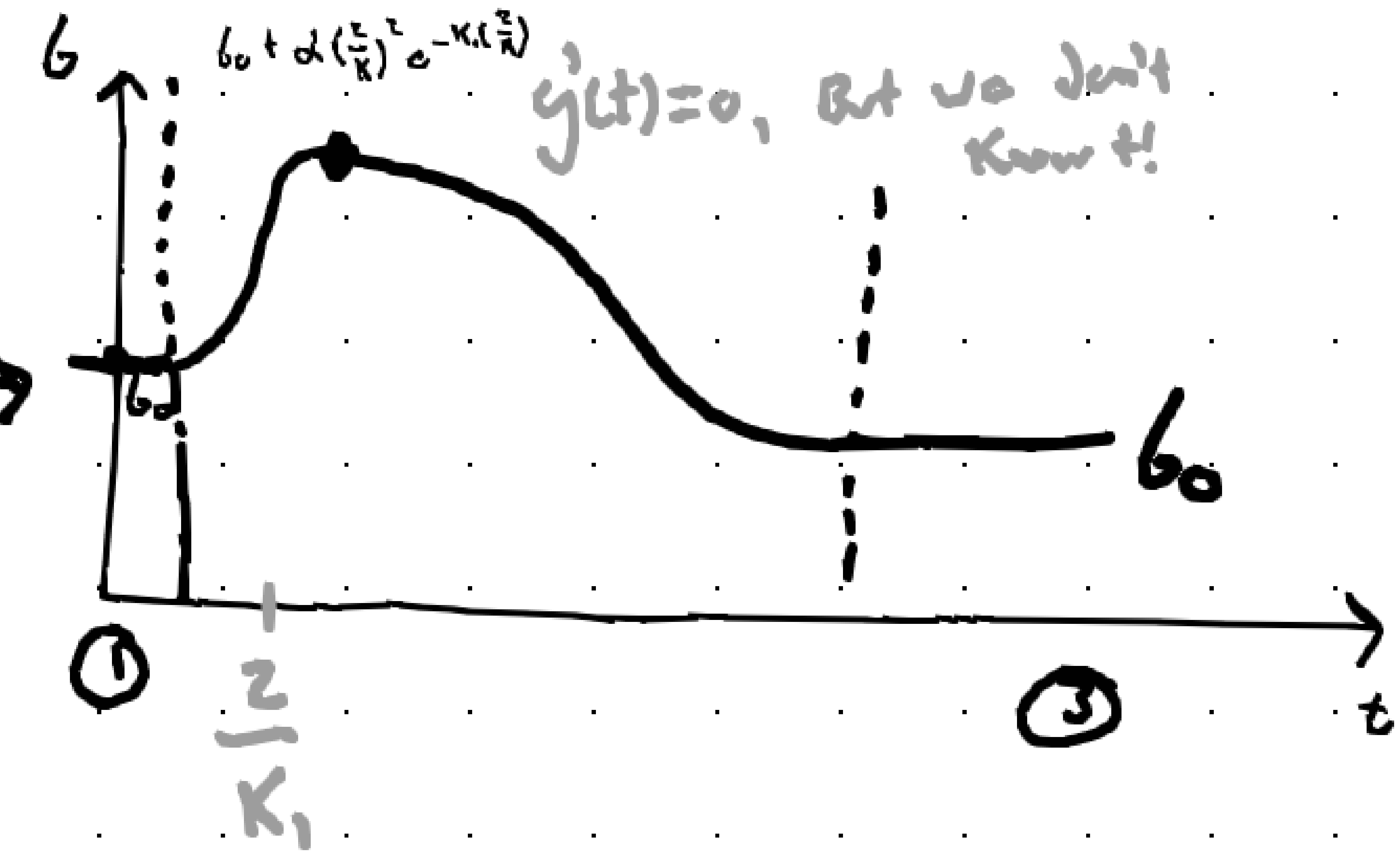
$$G(t) = G_0 + \alpha \underbrace{t^2}_{u} \underbrace{e^{-K_1 t}}_{v}$$

$$G'(t) = \alpha e^{K_1 t}(2t - K_1 t^2)$$

$$G'(0) = \alpha(1)(0-0)$$

$$G'(0) = 0$$

Initial Slope!



Medium Time

$$G'(t) = 0 \implies \underbrace{\alpha e^{K_1 t}}_{\text{We don't care about this!}} \underbrace{(2t - K_1 t^2)}_{\text{Solve t here.}} = 0$$

Find Zero!

$$\begin{cases} t_1 = 0 \\ t_2 = \frac{2}{K} \end{cases}$$

$$G\left(\frac{2}{K}\right) = G_0 + \alpha t^2 e^{-K_1 t} \qquad t(2 - K_1 t)$$

$$= G_0 + \alpha\left(\frac{2}{K}\right)^2 e^{-K_1\left(\frac{2}{K}\right)}$$

Large Time

$$G(t) = G_0 + \alpha\left(\frac{t^2}{e^{K_1 t}}\right) = \frac{2t}{K e^{Kt}} = \frac{2}{K^2 e^{Kt}}$$

Goes to Zero

$$G(t) = G_0$$

Sketching Cos and Sin

① Convert to Amplitude phase form!

Forms:

$$y(t) = A\sin(\omega t + \phi)$$ ← Pure Sinusoidal 

$$y(t) = A e^{-\alpha t}\cos(\omega t + \phi)$$ ← Damped 

② Find period (or for damped, pseudoperiod)

$$T = \frac{2\pi}{\omega}$$

ω = whatever the coefficient is inside cos or sin

i.e... $\sin(\overset{\downarrow}{2}t)$

③ Identify the envelope

Amplitude

$$y_{env}(t) \pm A e^{-\alpha t}$$

④ Start with your initial conditions

$$y(0) = y_0$$

$$\dot{y}(0) = v_0$$

Key Points
(optional)

1. Zero Crossing $\rightarrow y(t)=0 = t$
2. Max/min @ first period $y'(t)=0$
   $\hookrightarrow t$
   $\hookrightarrow y_{max/min}$
3. Intermediate points based on period.

Example

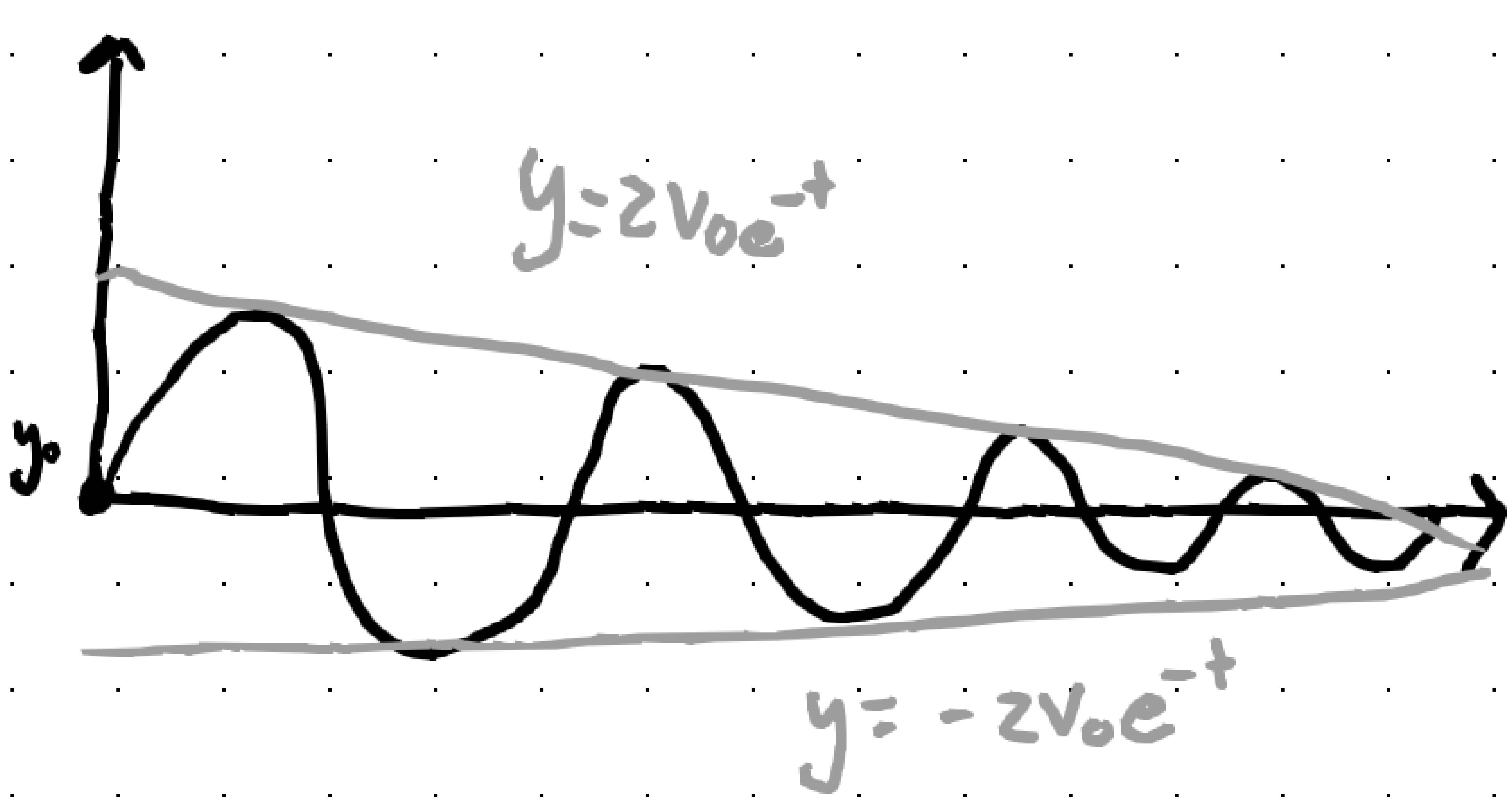
$$y(t) = \underbrace{2V_0}_{\text{Amp}}\, e^{-t} \sin \frac{t}{2}$$

$\dot{y}(0) = V_0 = \text{positive}$

$y(0) = 0$

$$T = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

$$y_{env} = \pm A e^{-\alpha t}$$
$$= \pm 2V_0 e^{-t}$$



Transfer Functions on LTI Systems

1- Convert to Amp Phase. We need Amplitude

$$H = \frac{\text{Steady State Amplitude}}{\text{Forcing Amplitude}}$$

"y_p" All Steady State Means

Example:

Convert to Amp phase

$$y_p = \frac{F_0}{(mg - m(\omega^2)^2 + (c\omega)^2} \left[(mg - m(\omega^2)\cos\omega t - (c\omega)\sin\omega t \right]$$

$$\frac{F_0 \sqrt{\cancel{(mg - m(\omega^2)^2 (+c\omega)^2}}}{\cancel{(mg - m(\omega^2))^2 + (c\omega)^2}} \sin(\omega t + \phi)$$

$$\text{Amp} = \frac{F_0}{\sqrt{mg - m(\omega^2)^2 + (c\omega)^2}}$$

$$H(\omega) = \frac{\text{Steady State Amp}}{\text{Forcing Amplitude}} = \frac{\frac{F_0}{\sqrt{mg - m(\omega^2)^2 + (c\omega)^2}}}{F_0}$$

$$H(\omega) = \frac{1}{\sqrt{mg - m(\omega^2)^2 + (c\omega)^2}}$$

Sketching Transfer function