$$\ddot{y} + 2\dot{y} + y = \cos(\omega t), \quad y(0) = 0, \quad \dot{y}(0) = 0$$

Find The Transfer Function

$\boxed{1}$ Step One: Find y_H

$$\lambda^2 + 2\lambda + \lambda = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-2 \pm \sqrt{4 - 4(1)(1)}}{2(1)}$$

* Critically Damped

$$\lambda_{1,2} = -1$$

$$(A + Bt)e^{-t}$$

$\boxed{2}$ Step Two: Find y_p

$$\lambda_{1,2} = -1$$

$$\ddot{y} + 2\dot{y} + y = \cos(\omega t)$$

$$y = RE[Y_p]$$

$$\ddot{Y}_p + 2\dot{Y}_p + Y_p = e^{i\omega t}$$

* No dupe

$$Y_p = De^{i\omega t}$$

$$\dot{Y}_p = i\omega De^{i\omega t}$$

$$\ddot{Y}_p = -\omega^2 De^{i\omega t}$$

$$D\cancel{e^{i\omega t}}(-\omega^2 + 2i\omega + 1) = \cancel{e^{i\omega t}}$$

$$D = \frac{1}{-\omega^2 + 2i\omega + 1}$$

$$Y_p = \left(\frac{1}{-\omega^2+2i\omega+1}\right)e^{i\omega t}$$

*clear up denominator

$$Y_p = \left(\frac{1}{-\omega^2+2i\omega+1}\right)e^{i\omega t} \cdot \frac{1-\omega^2-2i\omega}{\underbrace{1-\omega^2}_{a}\underbrace{-2i\omega}_{b}}$$

$= a^2 - b^2$

$$Y_p = \frac{1-\omega^2-2i\omega}{(1-\omega^2)^2+(2\omega)^2}e^{i\omega t}$$

$$Y_p = \frac{2i\omega+\omega^2-1}{(1-\omega^2)^2+4\omega^2}e^{i\omega t}$$

$$Y_p = \frac{2i\omega+\omega^2-1}{(1-\omega^2)^2+4\omega^2}(\cos\omega t + i\sin\omega t)$$

#Only Want Real Here...

☆

$$y_p = \frac{1}{(1-\omega^2)^2+4\omega^2}\Big((2i\omega)(i\sin\omega t) + (\omega^2)(\cos\omega t) - (1)(\cos\omega t)\Big)$$

$$y_p = \frac{1}{(1-\omega^2)^2+4\omega^2}\Big((-2\omega\sin\omega t) + (\omega^2\cos\omega t) - (\cos\omega t)\Big)$$

$$y_p = \frac{1}{(1-\omega^2)^2+4\omega^2}\Big((-2\omega\sin\omega t) + (\omega^2-1)\cos\omega t\Big)$$

$$y_p = \frac{1}{(1-\omega^2)^2 + 4\omega^2}\left((\underbrace{-2\omega}_{c_1}\sin\omega t) + (\underbrace{\omega^2-1}_{c_2})\cos\omega t\right)$$

prep For Transfer function — Amp Phase Form

$$A = \sqrt{c_1^2 + c_2^2}, \quad A = \sqrt{(-2\omega)^2 + (\omega^2-1)^2}$$

$$= \sqrt{\omega^4 + 2\omega^2 + 1}$$

$$\phi = \tan^{-1}\left(\frac{c_2}{c_1}\right)$$

cos form

c_1 and c_2 flipped for sin

$$\tan^{-1}\left(\frac{\omega^2-1}{-2\omega}\right)$$

$$y_p = \frac{1}{(1-\omega^2)^2 + 4\omega^2}\left(\sqrt{\omega^4+2\omega^2+1}\,\cos\left(\omega t - \tan^{-1}\left(\frac{\omega^2-1}{-2\omega}\right)\right)\right)$$

Ready for Transfer Function

$$H(\omega) = \frac{\text{Amp of Response}}{\text{Amp of forcing}} = \frac{\dfrac{\sqrt{\omega^4+2\omega^2+1}}{(\omega^4+2\omega+1}}{1}$$

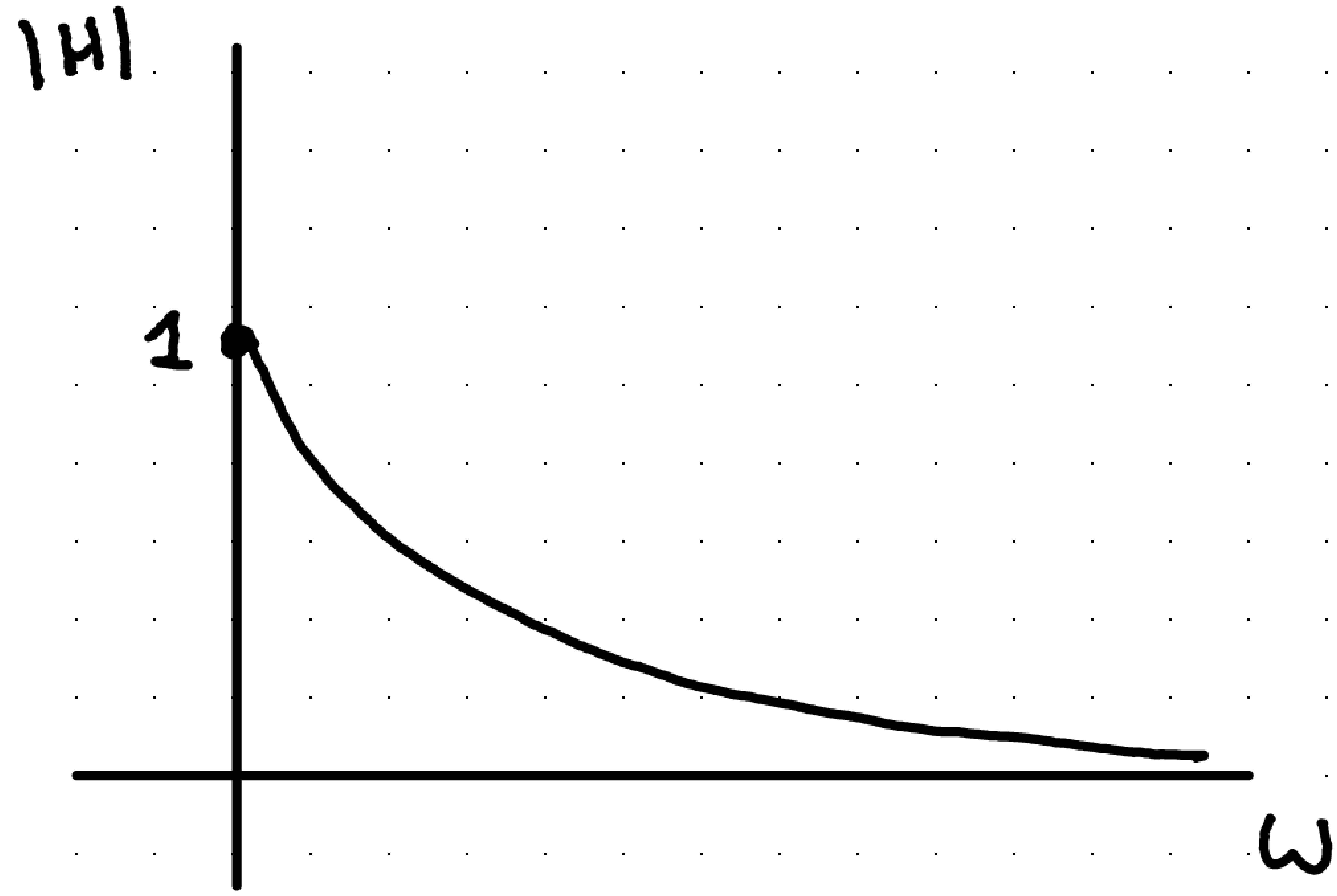
$\omega^4 - 2\omega^2 + 1 + 4\omega^2$

$\omega^4 + 2\omega^2 + 1$

$$\boxed{= \frac{1}{\sqrt{\omega^4+2\omega^2+1}}}$$

Graphing Transfer function:

$$\frac{1}{\sqrt{\omega^4 + 2\omega^2 + 1}}$$



Big Time goes to zero.